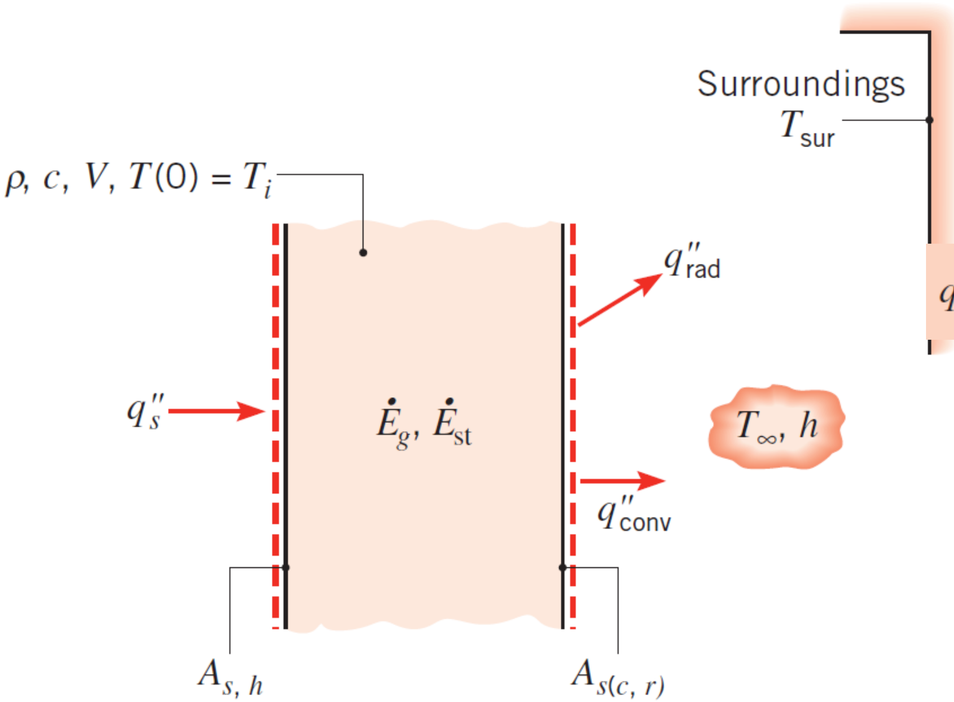
Transient Conduction

Transient Conduction

3. General Lumped Capacitance Analysis

- The figure below shows the general situation for which thermal conditions within a solid may be influenced simultaneously by convection, radiation, an applied surface heat flux, and internal energy generation.
- ***To study this system, the general Lumped Capacitance Analysis is needed.***



$$q''_s A_{s,h} + \dot{E}_g - (q''_{conv} + q''_{rad})A_{s(c,r)} = \rho V c \frac{dT}{dt}$$

$$q''_s A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \varepsilon\sigma(T^4 - T^4_{sur})]A_{s(c,r)} = \rho V c \frac{dT}{dt}$$

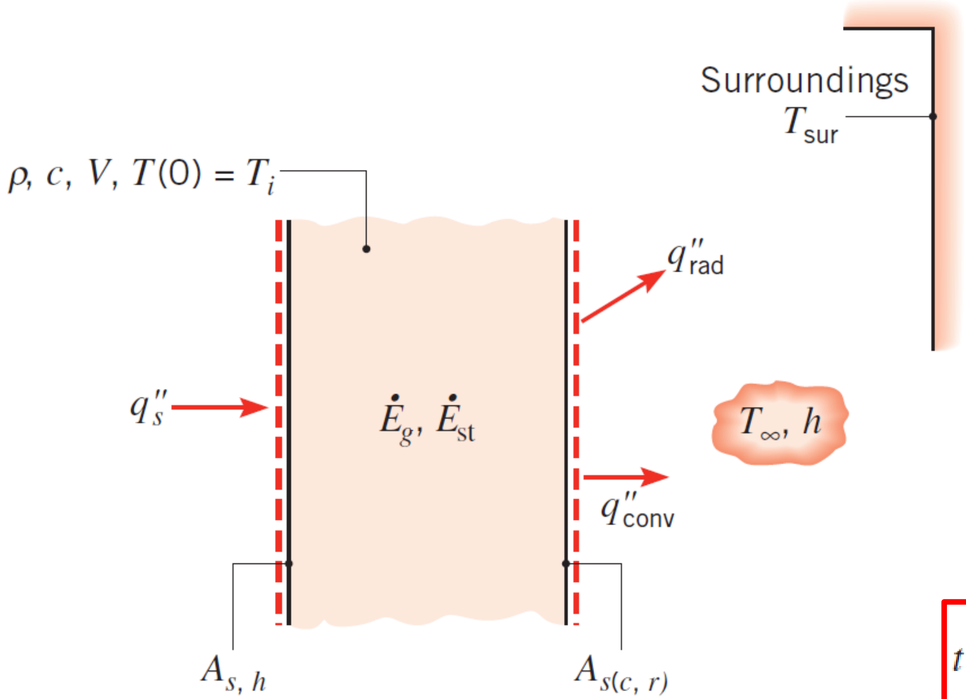
The transient conduction in a solid studied before, was initiated by convection heat transfer to or from an adjoining fluid.

Transient Conduction

3. General Lumped Capacitance Analysis

A. Only Radiation:

$$q_s'' A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \boxed{\varepsilon\sigma(T^4 - T_{sur}^4)]A_{s(c,r)}} = \rho V c \frac{dT}{dt}$$



$$\rho V c \frac{dT}{dt} = -\varepsilon A_{s,r}\sigma(T^4 - T_{sur}^4)$$

$$\frac{\varepsilon A_{s,r}\sigma}{\rho V c}\int_0^t dt = \int_{T_i}^{T} \frac{dT}{T_{sur}^4 - T^4}$$

$$t = \frac{\rho V c}{4\varepsilon A_{s,r}\sigma T_{sur}^3}\left\{\ln\left|\frac{T_{sur}+T}{T_{sur}-T}\right| - \ln\left|\frac{T_{sur}+T_i}{T_{sur}-T_i}\right| + 2\left[\tan^{-1}\left(\frac{T}{T_{sur}}\right) - \tan^{-1}\left(\frac{T_i}{T_{sur}}\right)\right]\right\}$$

$$\boxed{t = \frac{\rho V c}{3\varepsilon A_{s,r}\sigma}\left(\frac{1}{T^3} - \frac{1}{T_i^3}\right)}$$

Considering T_{sur}=0

Transient Conduction

3. General Lumped Capacitance Analysis

B. No Radiation:

$$q_s''A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \boxed{\varepsilon\sigma(T^4 - T_{\text{sur}}^4)]A_{s(c}}\,{}_{r)} = \rho V c \frac{dT}{dt}$$

$$\frac{d\theta}{dt} + a\theta - b = 0 \qquad a \equiv (hA_{s,c}/\rho V c) \qquad b \equiv [(q_s''A_{s,h} + \dot{E}_g)/\rho V c]$$

This equation is a linear, first-order, nonhomogeneous differential equation.

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}[1 - \exp(-at)]}$$

If b = 0, this equation can be reduced to:

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at)}$$

Transient Conduction

3. General Lumped Capacitance Analysis

C. Convection Only with Variable Convection Coefficient:

- In some cases, such as those involving free convection or boiling, the convection coefficient *h* varies with the temperature difference between the object and the fluid.
- The convection coefficient can often be approximated as follow:

$$h = C(T - T_\infty)^n$$

where n is a constant and the parameter C has units of $W/m^2.K^{(1+n)}$

$$-C(T - T_\infty)^n A_{s,c}(T - T_\infty) = -CA_{s,c}(T - T_\infty)^{1+n} = \rho V c \frac{dT}{dt}$$

$$\frac{\theta}{\theta_i} = \left[\frac{nCA_{s,c}\theta_i^n}{\rho V c} t + 1\right]^{-1/n}$$

Transient Conduction

3. General Lumped Capacitance Analysis

Exampel-3

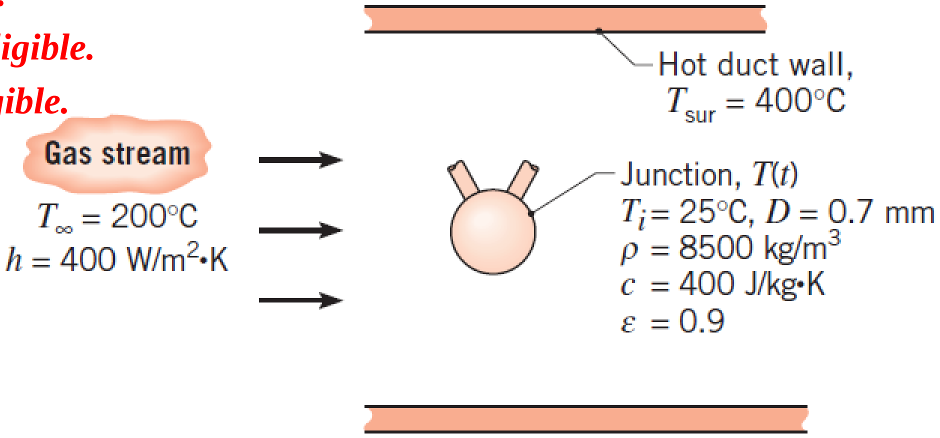
Consider the thermocouple and convection conditions in Example-1, but now allow for radiation exchange with the walls of a duct that encloses the gas stream. If the duct walls are at 400 °C and the emissivity of the thermocouple bead is 0.9, calculate the steady-state temperature of the junction. Also, determine the time for the junction temperature to increase from an initial condition of 25 °C to a temperature that is within 1 °C of its steady-state value.

1. Temperature of junction is uniform at any instant.

2. Radiation exchange with the surroundings is negligible.

3. Losses by conduction through the leads are negligible.

4. Constant properties.



Transient Conduction

3. General Lumped Capacitance Analysis

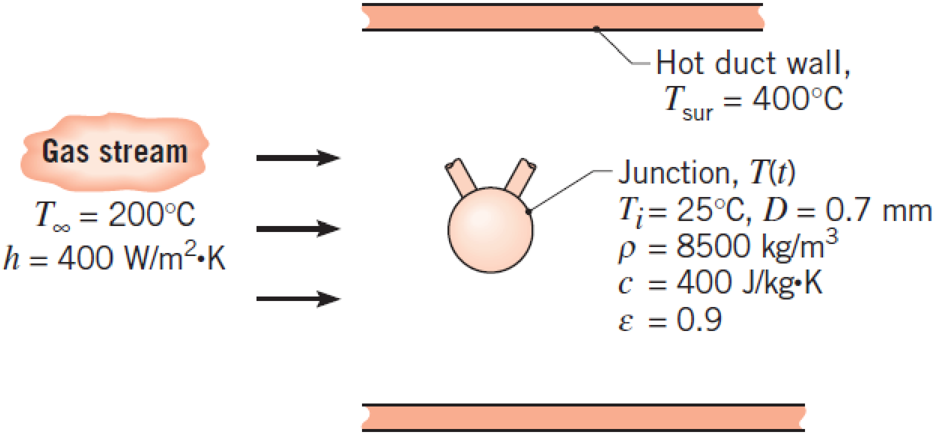
Exampel-3

For steady-state conditions, the energy balance on the thermocouple junction has the form

$$\dot{E}_{in} = 0$$

$$[\varepsilon\sigma(T_{sur}^4 - T^4) - h(T - T_\infty)]A_s = 0$$

$$T = 218.7°C$$

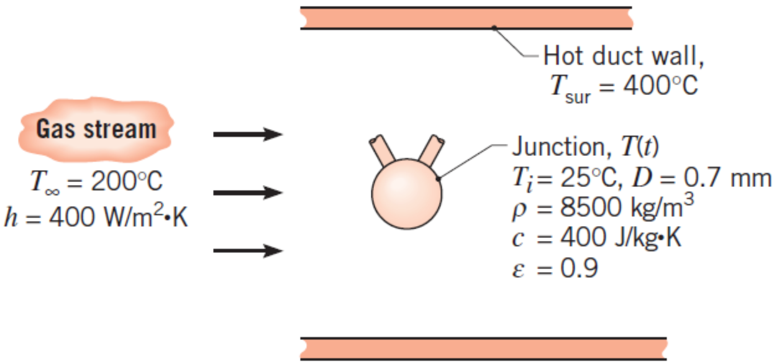


Transient Conduction

3. General Lumped Capacitance Analysis

Exampel-3

The temperature-time history, $T(t)$, for the junction, initially at $T_{(0)} = T_i = 25$ °C, follows from the energy balance for transient conditions



$$\dot{E}_{in} = \dot{E}_{st}$$

$$-[h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{sur}^4)]A_s = \rho V c \frac{dT}{dt}$$

The solution to this first-order differential equation can be obtained by numerical integration as follow:

$$T(4.9\text{ s}) = 217.7°\text{C}$$