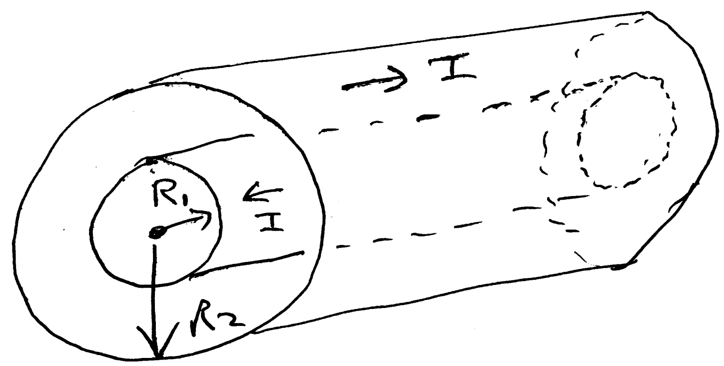
EM PROBLEM SET 6 2019

SOLUTIONS

1.



Apply Ampere's law

$$\oint_{\Gamma} \vec{B}(\vec{r}) \cdot d\vec{l} = \frac{(\text{current enclosed by loop } \Gamma)}{\varepsilon_0 c^2}.$$

We will choose Amperian loops to be circles of radius r,

$$\oint_{\Gamma} \vec{B}(\vec{r}) \cdot d\vec{l} = B(r) \cdot 2\pi r$$

(i) $r < R_1$, no current enclosed by the loop

$$\Rightarrow \quad B(r)\, 2\pi r = 0$$

$$\Rightarrow \quad \boxed{B(r) = 0}$$

(ii) $R_1 < r < R_2$,

current enclosed by circular loop $= I$

So $B(r)\, 2\pi r = \dfrac{I}{\varepsilon_0 c^2}$

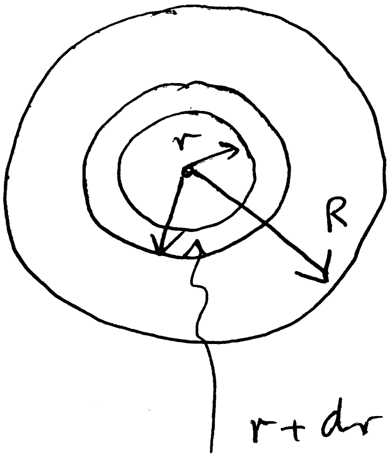
$$\Rightarrow \boxed{B(r) = \frac{I}{2\pi \varepsilon_0 c^2 r}}$$

(iii) $r > R_2$, current enclosed by loop $= 0$

$$\Rightarrow B(r)\, 2\pi r = 0$$

$$\Rightarrow \boxed{B(r) = 0}$$

2.



END ON VIEW OF WIRE.

$j(r) = \lambda r$ is the current density, or current per unit cross-sectional area.

The strip of wire between r and $r+dr$ has cross-sectional area $2\pi r \cdot dr$

$\Rightarrow$ current through the strip is

$$j(r)\, 2\pi r\, dr$$
$$= 2\pi \lambda r^2 dr$$

"Summing" over all strips between $r=0$ and $r=R$, the current in the wire is

$$I = \int_0^R 2\pi\lambda r^2 dr$$
$$= 2\pi\lambda \int_0^R r^2 dr$$
$$= 2\pi\lambda \left[\tfrac{1}{3} r^3\right]_0^R = \frac{2\pi\lambda}{3} R^3$$