PROBLEM SET 7 - 2019

1. Consider the electromagnetic wave

$$\vec{E}(\vec{r},t) = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{B}(\vec{r},t) = \vec{B}_0 \, e^{i(\vec{k}\cdot\vec{r}-\omega t)} .$$

In this question, we will examine the consequences of requiring that the Maxwell equation

$$\vec{\nabla} \times \vec{B}(\vec{r},t) = \frac{1}{c^2} \frac{\partial \vec{E}(\vec{r},t)}{\partial t} \qquad -(*)$$

is satisfied.

(a) Compute $\frac{\partial}{\partial x} \vec{B}(\vec{r},t)$.

(b) Using the result (a) and its generalization to $\frac{\partial}{\partial y} \vec{B}(\vec{r},t)$ and $\frac{\partial}{\partial z} \vec{B}(\vec{r},t)$, compute

$$\vec{\nabla} \times \vec{B}(\vec{r},t) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times \vec{B}(\vec{r},t)$$

(c) Show that the Maxwell equation (*) requires

$$\vec{k} \times \vec{B}_0 = -\frac{\omega}{c^2} \vec{E}_0$$

(d) Using $\frac{\omega}{|\vec{k}|} = c$ and $\frac{\vec{k}}{|\vec{k}|} = e_{\vec{k}}$ (unit vector in the direction of $\vec{k}$), show that the result in (c) is equivalent to

$$\vec{e_k} \times \vec{B_0} = -\frac{1}{c} \vec{E}_0$$

(e). On this diagram, draw the direction of $\vec{E}_0$ (given $\vec{B_0} \perp \vec{k}$):

(f) Given $\vec{B}_0 \perp \vec{k}$, determine the relation between $|\vec{E}_0|$ and $|\vec{B}_0|$.