Games

Surajeet Chakravarty

University of Exeter

Last week we looked at how individuals process new information to update their beliefs about a given process – Bayes' rule.

We also looked at a very simple model of search and briefly discussed the implications to reservation value

- Cost of search
- Risk aversion

We also briefly discussed the concept of bounded rationality and Simon's concept of satisficing.

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Today we are going to have a look at behavior in strategic settings.

We will start by looking at a particular class of games, *zero-sum* games

We will then move to a broader class of games and look at the fundamental building block of non-cooperative game theory: the Nash equilibrium.

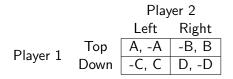
We will focus our attention at games where there is more than one equilibrium. A game is defined by three basic elements:

- ► A set of players: N
- A set of strategies: S
- A rule (or function), *F*, that maps strategies to outcomes.

Game theory's objective is to analyze stable outcomes of a game, given a set of preferences.

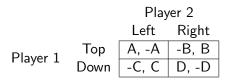
Early applications of game theory focused on games in which the total gains of all players on any given outcome would add up to zero.

These games are particularly useful to model conflict situations, where both parties cannot mutually agree on a satisfactory outcome.



Assume A, B, C, D > 0. Notice that in this case, neither Player 1 nor Player 2 have a preferred strategy:

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Assume A, B, C, D > 0. Notice that in this case, neither Player 1 nor Player 2 have a preferred strategy:

- If Player 1 picks Top with certainty, the best reply by Player 2 is to play Right
- If in turn Player 2 picks Bottom with certainty, the best reply by Player 2 is to play Left

In 1928, John von Neumann had a breakthrough in the analysis of zero-sum games when he proved that any zero-sum game with finitely many strategies has a *value*.

That is, Player 1 can come up with a strategy that guarantees him/her an expected payoff of V regardless of what Player 2 does, and vice versa.

In other words, both players can define a strategy that minimizes the maximum payoff the other player can achieve.

Zero-Sum Games, the Minimax Theorem and Mixed Strategy Nash Equilibrium

As it turns out, in Zero-Sum Games, Minimax strategies coincide with mixed strategy Nash equilibrium strategies

 Mixed strategy Nash equilibria are easier to compute, so we'll focus on those

To find the equilibrium of a zero-sum game, we must ask each player to assign a probability to each action available to him/her.

- Let p be Pr(Top), 1 p be Pr(Bottom)
- Let q be Pr(Left), 1 q be Pr(Right)

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Zero-Sum Games, the Minimax Theorem and Mixed Strategy Nash Equilibrium

Player 1 will choose p to make Player 2 indifferent between playing Left and Right:

$$\blacktriangleright E(Left) = p \times (-A) + (1-p) \times (C)$$

$$\blacktriangleright E(Right) = p \times (B) + (1 - p) \times (-D)$$

•
$$E(Left) = E(Right) \iff p \times (-A) + (1-p) \times (C) =$$

 $p \times (B) + (1-p) \times (-D)$
• $p = \frac{C+D}{A+B+C+D}$

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Zero-Sum Games, the Minimax Theorem and Mixed Strategy Nash Equilibrium

Player 2 will choose q to make Player 1 indifferent between playing Top and Bottom:

$$\blacktriangleright E(Top) = q \times (A) + (1-q) \times (-B)$$

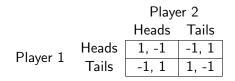
•
$$E(Bottom) = q \times (-C) + (1-q) \times (D)$$

►
$$E(Top) = E(Bottom) \iff q \times (A) + (1 - q) \times (-B) =$$

 $q \times (-C) + (1 - q) \times (D)$
► $p = \frac{B+D}{A+B+C+D}$

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An example: The Matching Pennies Game



Rules:

- Both players simultaneously name one side of a coin
- Player 1 wins if both players name the same side of the coin
- Player 2 wins if the two players name different sides

The equilibrium of the MP game is for both players to pick each side of the coin with equal probability.

• That is, p = 1/2 and q = 1/2.

The value of the MP game is the expected payoff each player gets by playing the equilibrium strategy:

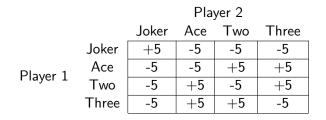
O'Neill (1987) proposed a very simple experiment to test the theory of zero-sum games.

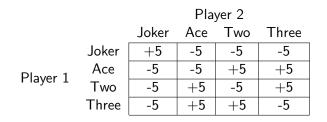
- ▶ The experiment was played by 50 students working in 25 pairs
- Subjects sat opposite each other at a table (no anonymity)
- Each subject held four cards: Ace, 2, 3 and a Joker.
- ► Each player started the experiment with \$2.50 in 5 cent coins.

Each round of the experiment worked as follows:

- When prompted, subjects picked a card and placed it face down on the table.
- Following another prompt, subjects then turned the cards over and determined the winner.
- The winner collected 5 cents from the loser, and they moved to the next round.

The following matrix displays the payoffs to Player 1 (row player). The payoffs to Player 2 are the negative of the payoff for player 1.

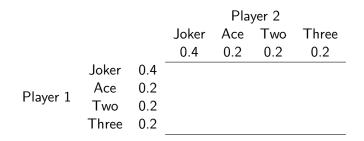




Player 1 wins if both players pick the Joker or if they both pick different numbered cards (the Ace counts as a 1)

Player 2 wins if both players pick the same numbered card or if Player 1 picks the Joker and Player 2 picks any other card.

Equilibrium strategy in the O'Neill game



Note that the equilibrium of this game is invariant to preferences

- There are only two outcomes in this game: a win or a loss...
- ... and gains are preferred to losses.

Note also the clever use of the Joker strategy, which ensures that the equilibrium strategy is asymmetric

Equilibrium strategy in the O'Neill game

		С	olumn Pla	ayer Choi	ice	Marginal Frequencies For Row Player:
		1	2	3	J	
		.044	.043	.043	.091	.221
	1	(.040)	(.040)	(.040)	(.080)	(.200)
		[.004]	[.004]	[.004]	[.005]	[.008]
		.046	.038	.038	.092	.215
	2	(.040)	(.040)	(.040)	(.080)	(.200)
Row		[.004]	[.004]	[.004]	[.005]	[.008]
Player						
Choice		.049	.032	.037	.085	.203
	3	(.040)	(.040)	(.040)	(.080)	(.200)
		[.004]	[.004]	[.004]	[.005]	[.008]
		.086	.065	.051	.158	.362
	J	(.080)	(.080)	(.080)	(.160)	(.400)
		[.005]	[.005]	[.005]	[.007]	[.010]
Marginal Fr	equencies	.226	.179	.169	.426	
for Column		(.200)	(.200)	(.200)	(.400)	
		[.008]	[.008]	[.008]	[.010]	

TABLE I

RELATIVE FREQUENCIES OF CARD CHOICES IN O'NEILL'S EXPERIMENT^a

^a Numbers in parentheses represent minimax predicted relative frequencies. Numbers in brackets represent standard deviations for observed relative frequencies under the minimax hypothesis.

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The choice frequencies in the experiment match closely those of theory.

- Player 1s chose the Joker 36.2% of the time (0.362 = 0.400, t = 0.051)
- Player 2s chose the Joker 43.0% of the time (0.430 = 0.400, t = 0.150)

O'Neill also argues that the frequency of choices of numbered cards (Ace, Two and Three) is pretty close to theory:

- Player 1s chose 578 aces, 565 twos and 532 threes
- Player 2s chose 593 aces, 470 twos and 446 threes
- ► Only the P2s distribution was significantly different from the prediction of equal probabilities using a χ^2 test.
- O'Neill argued that this was probably due to the Ace being a "loaded" card.

O'Neill finally looks at winning probabilities in the data.

The theory predicts that:

- ► The row player should win 40% of the time
- ► The column player should win 60% of the time

The data:

- ► The row players won 40.1% of the time
- ► The column players won 59.9% of the time

While the data looks close to Minimax play, Brown and Rosenthal (1990) re-evaluate O'Neill's data, and show that in fact Minimax can be confidently ruled out.

There are two issues to consider. The first is that although the choice frequencies are close to prediction, they may be based on a very different decision model.

For example, suppose Player 1 believes Player 2 thinks Player 1 picks a card at random 20% of the time. The other 80% of the time, Player 1 actually randomizes.

Lies, Damn Lies and Statistics

Let's work out the optimal behavior for Player 2 if (s)he believes Player 1 is randomizing:

$$E_2(Joker) = \frac{1}{4}(-5+5+5+5) = 4$$
$$E_2(Ace) = \frac{1}{4}(+5+5-5-5) = 0$$
$$E_2(Two) = \frac{1}{4}(+5-5+5-5) = 0$$
$$E_2(Three) = \frac{1}{4}(+5-5-5+5) = 0$$

In short, if Player 2 believes Player 1 picks cards at random, (s)he would always pick the Joker card.

So, what would Player 1 do if (s)he believes Player 2 is picking the Joker card?

So, if Player 1 believes Player 2 thinks Player 1 picks a card at random 20% of the time, he will pick the Joker 20% of time.

If Player 1 picks at random the rest of the time, here's what the strategy of Player 1 looks like:

▶ 0.4, 0.2, 0.2, 0.2 for Joker, Ace, Two and Three.

This is indistinguishable from Minimax!

So, when testing for the null hypothesis of Minimax play, we need to use more stringent tests than looking at the frequencies of play of a particular card.

Brown and Rosenthal start by running a Chi-Squared test on the joint distribution of play by both sets of players against the predicted Minimax.

• They rejected Minimax play at p < 0.01

But what about the winning probability? Surely that is the key piece of evidence?

Well, the problem is that a lot of (random) behavior will give very similar probabilities of winning.

If both players just pick at random, the row player will win with 43% probability!

Lies, Damn Lies and Statistics: Looking at the data pair-by-pair

	Winning % for		Row Player Choice				Column Player Choice				
Pair #	Row Player	1	2	3	J	1	2	3	J	Comments	
1	.391	.257	.286*	.276	.181*	.229	.210	.210	.352	a, c	
2 3	.295	.171	.181	.210	.438	.152	.152	.143	.552*	ъ	
3	.390	.162	.114*	.181	.543*	.219	.133	.095*	.552*	a, b, c	
4	.419	.219	.267	.181	.333	.067*	.086*	.124	.724*	b,c	
5	.343	.171	.171	.190	.467	.324*	.086*	.143	.448	b,c	
6	.419	.257	.143	.210	.390	.257	.200	.095*	.448	ь	
7	.476	.238	.229	.229	.305*	.219	.190	.238	.352	_	
8	.467	.171	.286*	.219	.324	.267	.248	.190	.295*		
9	.362	.181	.257	.267	.295*	.257	.181	.219	.343	_	
10	.390	.257	.171	.152	.419	.200	.190	.200	.410	_	
11	.390	.276	.248	.171	.305*	.229	.200	.200	.371		
12	.543	.276	.133	.105*	.486	.210	.200	.162	.429	a, c	
13	.410	.219	.267	.248	.267*	.114*	.219	.133	.533*	a, b, c	
14	.467	.267	.229	.200	.305*	.267	.248	.257	.229*	b, c	
15	.324	.200	.181	.162	.457	.295*	.143	.190	.371	_	
16	.343	.152	.248	.162	.438	.219	.238	.162	.381	_	
17	.362	.200	.219	.219	.362	.229	.171	.190	.410		
18	.486	.238	.181	.190	.390	.219	.152	.219	.410		
19	.390	.286*	.190	.200	.324	.171	.162	.162	.505		
20	.438	.210	.219	.143	.429	.210	.171	.124	.495		
21	.476	.190	.229	.210	.371	.276	.219	.181	.324		
22	.400	.200	.162	.181	.457	.286*	.181	.190	.343		
23	.448	.229	.286*	.324*	.162*	.295*	.181	.105*	.419	a, b, c	
24	.495	.248	.257	.238	.257*	.248	.162	.219	.371	a	
25	.333	.238	.229	.200	.333	.181	.152	.076*	.590*	b,c	

TABLE II

RELATIVE FREQUENCIES OF CARD CHOICES AND ROW-PLAYER WINS IN O'NEILL'S EXPERIMENT BY PLAYER PAIR

* Denotes rejection (at .05 level) of minimax binomial model for a given card.

^a Denotes joint rejection (at .05 level) of minimax multinomial model for all cards chosen by the row player, based on Pearson statistic and $\chi^2(3)$.

^b Denotes joint rejection (at .05 level) of minimax multinomial model for all cards chosen by the column player, based on Pearson statistic and $\chi^2(3)$.

^{*} Denotes joint rejection (at .05 level) of minimax multinomial model for all cards chosen by both players, based on Pearson statistic and $\chi^2(6)$.

Lies, Damn Lies and Statistics: Looking at the data individual-by-individual

TABLE IV

Results of Significance Tests from Logit Equations for the Choice of a Joker Card

	Estimating Equation ^a : J =	$\begin{array}{l} G[a_0+a_1 \log(J)+a_2 \log(J)+b_0 J^{\star}+b_1 \log(J^{\star})+b_2 \log(J^{\star})\\ +c_1 \log(J) \log(J^{\star})+c_2 \log(J) \log(J^{\star})] \end{array}$
	Null Hypothesis	Player Pairs Whose Behavior Allows Rejection of the Null Hypothesis at the .05 Level
(1)	$a_1, a_2, \\ b_0, b_1, b_2, \\ c_1, c_2 \text{ all } = 0$	Row: 2, 5, 7, 8, 10, 11, 12, 14, 16, 17, 20, 21, 22 Column: 2, 4, 6, 7, 8, 9, 10, 11, 12, 14, 17, 18, 19, 20, 21, 23, 24, 25
(2)	$a_1, a_2 = 0$	Row: 6, 7, 8, 10, 12, 17, 21, 22 Column: 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 20, 23, 24, 25
(3)	$b_1,b_2,c_1,c_2 \text{ all}=0$	Row: 4, 5, 7, 8, 10, 11, 12, 14, 16, 17, 21, 22, 23, 25 Column: 1, 2, 17, 18, 19, 21, 24, 25
(4)	$c_1, c_2 = 0$	Row: 8, 9, 10, 11, 12, 14, 25 Column: 2, 6, 9, 17, 21, 24, 25
(5)	$b_1, b_2 = 0$	Row: 4, 5, 7, 10, 12, 14, 16, 17, 21, 23, 25 Column: 1, 2, 17, 19, 21, 25
(6)	$b_0 = 0$	Row: 2,4 Column: 2,4

^a The symbols J and J^{*} denote the choice of a joker card by a player and by his opponent, respectively. The function G[x] denotes the function $\exp(x)/[1 + \exp(x)]$. Rejections are based on likelihood-ratio tests.

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This experiment is a very good example of the methodological problems faced by experimental economists:

- Subjects require repetition in order to learn how to play the game.
- But repetition introduces dynamic effects, particularly if you are playing the same person/people every time.
- One-shot decisions may not always help, since in the O'Neill game, what you want to measure is a probability (1 obs per subject doesn't give you much power)

Alternative: Use Experts!

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Palacios-Huerta (2003) studies 1417 penalty kicks taken in professional games in European competitions.

Distribution of strategies and scoring rates											
Score difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	Scoring rate
0	580	16.9	1.3	21.0	4.3	0.8	5.6	19-4	0.6	27.9	81.9
1	235	19.1	0	19-1	4.2	0	2.5	28.0	0	26.8	77.8
-1	314	19.7	0.9	25.8	1.9	0	6.4	20.0	0.6	30.2	80.2
2	97	23.7	2.0	17.5	5.2	0	0	20.6	1.0	29.9	75.2
-2	114	26.3	0	25.4	3.5	0	3.5	16.6	0	24.5	78-0
3	27	14.8	0	18.5	3.7	0	11-1	22.2	0	29.6	77.7
-3	23	30.4	0	30.4	0	0	0	21.7	0	17-4	82.6
4	7	42.8	0	28.5	0	0	0	14.2	0	14.2	100
-4	12	25·0	0	25.0	0	0	16.6	16.6	0	16.6	83.3
Others	8	50.0	0	0	0	0	12.5	37.5	0	0	87.5
Penalties shot in:											
First half	558	21.1	0.8	19.8	3.9	0.3	3.5	20.0	0.3	29.7	82.9
Second half	859	18.7	0.9	23.2	3.3	0.3	3.6	22.8	0.5	26.3	78.3
Last 10 min	266	21.8	0	21.0	0.3	0	0.7	25.1	0	30.8	73.3
All penalties Scoring rate	1417 80-1	19-6 55-2	0.9 100.0	21.9 94.2	3.6 94.1	0-3 50-0	3.6 82.3	21.7 96.4	0.5 100.0	27.6 71.1	80-1

TABLE 1 Distribution of strategies and scoring rates

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the 🚊 🕨 🥃 🕤

Surajeet Chakravarty

Games

					Left	footed k	ickers				
Score											Scoring
difference	#Obs.	LL	LC	LR	CL	CC	CR	RL	RC	RR	rate
0	174	17.8	1.7	20.1	6.3	0	8.6	22.9	0.5	21.8	8 2·7
1	73	28.7	0	30.1	4.1	0	2.7	19-1	0	15.0	78.0
-1	92	29.3	1.0	26.0	1.0	0	2.0	21.7	1.0	18.4	82.6
2	29	51.7	0	13.7	3.0	0	0	10.3	0	20.6	72.4
-2	30	40.0	0	13.3	3.0	0	3.0	20.0	0	20.0	76.6
All penalties	406	29.3	1.4	20.4	4.4	0	3.9	23.8	0	16.5	
Scoring rate	81.0	62·1	100	95·1	94.4	0	81 ·2	93.8	0	61 ·2	
					Right	-footed k	ickers				
0	406	16.4	1.2	21.4	3.4	1.2	4.4	20.4	0.7	30.5	83-2
1	162	14.8	0	14.2	4.3	0	2.4	32-1	0	32.1	77.7
-1	222	15.7	1.0	25.6	2.2	0	0	19.3	1.0	35-1	80.6
2	68	11.7	2.9	19.1	5.8	0	0	25.0	1.4	33.8	76-4
-2^{-2}	84	21.4	0	29.7	3.5	0	3.5	15.4	0	26.2	78.5
All penalties	1011	15-8	0.6	22.5	3.2	0.5	3.4	20.8	0.6	32.1	
Scoring rate	79-8	50.0	100	93.8	93.9	60.0	82.8	97.6	100	73.2	

TABLE 2

Distribution of strategies and scoring rates by kicker type

Note: The first letter of the strategy denotes the kicker's choice and the second the goalkeeper's choice. "R" denotes the R.H.S. of the goalkeeper, "L" denotes the L.H.S. of the goalkeeper, and "C" denotes centre.

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		Mixt	ure	Scoring	rates	Pearson	<i>p</i> -value	
Player	#Obs.	L	R	L	R	statistic		
Kicker 1	34	0-32	0.68	0.91	0-91	0.000	0.970	
Kicker 2	31	0-35	0.65	0.82	0.80	0.020	0.902	
Kicker 3	40	0-48	0.52	0.74	0.76	0.030	0.855	
Kicker 4	38	0-42	0.58	0.88	0.91	0.114	0.735	
Kicker 5	38	0-50	0.50	0.79	0.84	0.175	0.676	
Kicker 6	36	0.28	0.72	0.70	0.77	0.185	0.667	
Kicker 7	41	0.20	0.80	0.75	0.82	0.191	0.662	
Kicker 8	35	0.31	0.69	0.82	0.75	0.199	0.656	
Kicker 9	31	0-19	0.81	0.83	0.92	0-416	0.519	
Kicker 10	35	0.37	0.63	0.86	0.77	0.476	0.490	
Kicker 11	32	0-48	0.52	0.87	0.94	0.521	0.471	
Kicker 12	32	0-48	0.52	0.87	0.94	0.521	0.471	
Kicker 13	38	0-55	0.45	0.76	0.88	0.907	0.341	
Kicker 14	30	0.33	0.67	0.90	0.75	0.938	0.333	
Kicker 15	30	0-50	0.50	0.80	0.93	1-154	0.283	
Kicker 16	42	0-43	0.57	0.89	0.75	1.287	0.257	
Kicker 17	40	0-42	0.58	0.58	0.85	1-637	0.201	
Kicker 18	46	0-44	0.56	0.90	0.77	1-665	0.197	
Kicker 19	39	0.48	0.52	0.74	0.90	1.761	0.184	
Kicker 20	40	0.35	0.65	0.93	0.69	2.913	0.088*	
Kicker 21	40	0.42	0.58	0.65	0.91	4-322	0.038*	
Kicker 22	40	0-40	0.60	1.00	0.75	4-706	0.030*	
All kickers	808	0-3998	0.6002	0.8111	0.8268			
Goalkeeper 1	37	0-38	0.62	0.21	0.22	0.000	0.982	
Goalkeeper 2	38	0-39	0.61	0.20	0.22	0.017	0.898	
Goalkeeper 3	30	0.60	0-40	0.28	0.25	0.028	0.866	
Goalkeeper 4	50	0-46	0.54	0.17	0.15	0.061	0.804	
Goalkeeper 5	36	0.33	0.67	0.25	0.21	0.080	0.777	
Goalkeeper 6	34	0.44	0.56	0.27	0.21	0.147	0.702	
Goalkeeper 7	37	0.19	0.81	0.14	0.10	0.221	0.638	
Goalkeeper 8	37	0-54	0-46	0.25	0.18	0.293	0.588	
Goalkeeper 9	32	0-56	0-44	0.22	0-14	0-326	0.568	
Goalkeeper 10	40	0-45	0.55	0.11	0-18	0-388	0.533	
Goalkeeper 11	33	0.18	0.82	0.17	0.30	0.416	0.519	
Goalkeeper 12	30	0.27	0.73	0.25	0.14	0.545	0.460	
Goalkeeper 13	34	0.41	0.59	0.14	0.25	0.578	0-447	
Goalkeeper 14	40	0-50	0.50	0.15	0.25	0.625	0.429	
Goalkeeper 15	44	0-45	0.55	0.10	0.21	0.957	0.328	
Goalkeeper 16	36	0-31	0-69	0.09	0.24	1.804	0.298	
Goalkeeper 17	42	0.55	0-45	0.30	0.11	2-449	0.118	
Goalkeeper 18	42	0-38	0.62	0.13	0-35	2-506	0.113	
Goalkeeper 19	42	0-40	0-60	0.35	0.12	3-261	0.071*	
Goalkeeper 20	40	0.60	0.40	0.08	0.37	5-104	0.024*	
All goalkeepers	754	0.4231	0.5769	0.1943	0.2068			

TABLE 3 Tests for equality of scoring probabilities

Note: *Indicates rejected at 10% level, and **indicates rejected at 5% level.

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	-			-		
		bserva		Runs		
Player	L	R	Total	R	$\Phi[f(r-1; s)]$	$\Phi[f(r;s)]$
Kicker 1	11	23	34	16	0-439	0-597
Kicker 2	11	20	31	21	0.983**	0.994
Kicker 3	19	21	40	22	0-570	0.691
Kicker 4	16	22	38	19	0.365	0.496
Kicker 5	19	19	38	22	0.689	0.795
Kicker 6	10	26	36	15	0.344	0.509
Kicker 7	8	33	41	14	0-423	0-625
Kicker 8	11	24	35	15	0.263	0.407
Kicker 9	6	25	31	9	0-097	0.241
Kicker 10	13	22	35	19	0-599	0-729
Kicker 11	15	17	32	19	0.714	0-822
Kicker 12	15	17	32	20	0.822	0.901
Kicker 13	21	17	38	23	0.816	0.891
Kicker 14	10	20	30	12	0.117	0.221
Kicker 15	15	15	30	18	0.711	0.824
Kicker 16	18	24	42	19	0.164	0.254
Kicker 17	19	21	40	20	0.321	0.443
Kicker 18	20	26	46	19	0.693	0.789
Kicker 19	19	20	39	19	0.259	0.374
Kicker 20	14	26	40	14	0.022	0.049*
Kicker 21	17	23	40	18	0-159	0.251
Kicker 22	16	24	40	22	0-668	0-779
Goalkeeper 1	14	23	37	17	0-249	0.374
Goalkeeper 2	15	23	38	21	0-678	0-790
Goalkeeper 3	18	12	30	12	0.065	0-130
Goalkeeper 4	23	27	50	24	0.250	0.350
Goalkeeper 5	12	24	36	17	0.424	0.576
Goalkeeper 6	15	19	34	15	0.124	0.212
Goalkeeper 7	7	30	37	13	0-533	0-738
Goalkeeper 8	20	17	37	20	0.516	0.647
Goalkeeper 9	18	14	32	19	0-739	0.842
Goalkeeper 10	18	22	40	14	0.009	0.021**
Goalkeeper 11	6	27	33	11	0.423	0.661
Goalkeeper 12	8	22	30	15	0-802	0.908
Goalkeeper 13	14	20	34	19	0.644	0.767
Goalkeeper 14	20	20	40	22	0.564	0.685
Goalkeeper 15	20	24	44	27	0-871	0.925
Goalkeeper 16	11	25	36	16	0.378	0.535
Goalkeeper 17	23	19	42	28	0.964*	0.983
Goalkeeper 18	16	26	42	23	0.713	0.814
Goalkeeper 19	17	25	42	18	0.113	0.187
Goalkeeper 20	24	16	40	19	0.285	0-408

TABLE 5 Tests of serial independence of choices

Note: *Indicates rejected at 10% level, and **indicates rejected at 5% level.

Surajeet Chakravarty

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Results of significance tests from logit equations for the choice of the natural side

 $\begin{aligned} \text{Estimating equation:} \\ R &= G[a_0 + a_1 \text{lag}(R) + a_2 \text{lag}2(R) + b_0 R^* + b_1 \text{lag}(R^*) + b_2 \text{lag}2(R^*) + c_1 \text{lag}(R) \text{lag}(R^*) + c_2 \text{lag}2(R) \text{lag}2(R^*)] \end{aligned}$

			llows rejection s at the:	
Null hypothesis:		0.05 level	0.10 level	0.20 level
$1. a_1 = a_2 = b_0 = b_1 = b_2 = c_1 = c_2 = 0$	Kicker	_	2	2,18
	Goalkeeper	-	7	7,15
$2. a_1 = a_2 = 0$	Kicker	_	2	2,14
	Goalkeeper	_	8	8,17
$3. b_1 = b_2 = 0$	Kicker	-	_	5
	Goalkeeper	_	7	7
$4. c_1 = c_2 = 0$	Kicker	_	_	6
	Goalkeeper	_	_	14
$5. b_0 = 0$	Kicker	_	11,17	5, 11, 17, 21
•	Goalkeeper	-	3,16	3, 9, 10, 16

Notes: R and R* denote the choice of "natural" strategy by a kicker and a goalkeeper, respectively (right for a right-footed kicker and for a goalkeeper facing a right-footed kicker and for a goalkeeper facing a left-footed kicker. The terms "lag" and "lag2" refer to the strategies previously followed in the ordered sequence of penalty kicks. C(1) denotes the function exp(1)/(1 + exp(x)). Rejections are based on likelihood-ratio tests.

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Experienced players may be less prone to behavioral biases/more rational.

- Higher stakes
- They don't face the same opponent in consecutive penalty kicks

Game theory is interested in finding outcomes from which players have no incentive to deviate.

 i.e. outcomes in which my actions are optimal given what the other players are doing (and vice versa).

Such an outcome is a Nash Equilibrium (NE) of a game.

Some games have a unique NE; others have many NE.

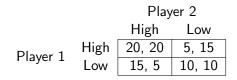
Take two workers operating in a factory.

Their payoff is a function of joint output

However each worker has a private cost of effort

 $N = \{1, 2\}$

 $S_i = \{High, Iow\}$



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There are two Nash equilibria in pure strategies:

- (High, High)
- (Low, Low)

In this game, strategies are strategic complements:

Player 2's best response to a rise (drop) in player 1's action is a rise (drop) in his action.

Which equilibrium should be played?

This particular type of game is interesting to economists as it captures the idea of externalities:

- Team production processes (e.g. min. effort game);
- Industrial Organisation (e.g. market entry games);

It is important to understand why would a set of agents be stuck in bad equilibria.

Is this due to strategic or behavioural reasons?

Common criteria for equilibrium selection:

Focal points;

Payoff dominance;

Risk dominance.

Thomas Schelling proposed a class exercise to his students.

They had to select a time and a place to meet up in New York city the following day.

The majority of his students chose Grand Central Station at 12 noon.

Certain equilibria are "intuitive" or naturally salient and as a result get chosen more often.

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Focal points: Crawford et al. (2008)

This paper studies the extent to which salience of decision labels could lead to resolution of the coordination problem.

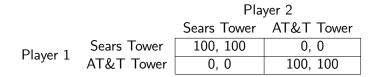
In pilot data, they modified Schelling's example and set up a simple coordination game

University of Chicago students had to choose to meet in one of two locations:

- The Sears Tower, a landmark Chicago building;
- The AT&T Tower, a little known building across the street from the Sears Tower.

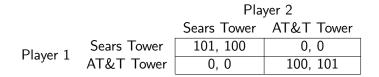
They considered three conditions

Symmetric Treatment:



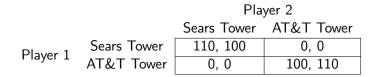
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Slightly Asymmetric Treatment:



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Asymmetric Treatment:



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The percentage of subjects who chose "Sears Tower" is as follows:

Treatment		High Payoff	Low Payoff
Symmetry	90% (n=60)		
Slight Asymmetry		58% (n=50)	61% (n=49)
Asymmetry		47% (n=30)	50% (n=28)

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Expected coordination rates were equal to:

- ► Symmetry: 82%
- Slight Asymmetry: 52%
- Asymmetry: 50%
- Mixed Strategy Nash Equilibrium: 50%!

The mere presence of small payoff asymmetries dramatically reduces the power of focal points (in Crawford et al.'s data set).

Payoff dominance is a relatively intuitive concept;

If an equilibrium is Pareto superior to all other NE, then it is payoff dominant.

An outcome Pareto-dominates another if all players are at least as well off and at least one is strictly better off.

It is intuitively appealing, but the data does not seem to fully support it.

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The concept of risk dominance is based upon the idea that a particular equilibrium may be riskier than another.

This is NOT related to concavity of the utility function!!!

In simple 2x2 games, RD could be thought as how costly are deviations from a particular equilibrium vis--vis the other?

There is no general way to compute a risk dominant equilibrium in n × n games.

Although rational agents ought to follow payoff dominance, experimental data shows subjects often play the risk dominant ("safer") equilibrium.

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Larger N

The concept of coordination centers around beliefs

The choice of equilibrium will depend on what you think the other player will do.

The more players there are, the harder it is to coordinate: it is harder to form consistent beliefs about every players action — it only takes one player to destroy the equilibrium.

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Incentive structure

Are equilibria unfair?

Are there focal points?

Can players communicate?

Run a simple coordination experiment.

Vary the extent subjects can communicate with one another:

- No communication;
- One-way (non-binding) announcements;
- Two-way (non-binding) announcements.

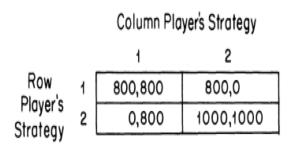


FIGURE II

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Cooper et al. (1992)

TABLE I SCG						
		Strategy				
		1		2		
Announcements						
No communication:				_		
One-way:						
Rep. 1&3		19		91		
Rep. 2		2		53		
Total		21		144		
Two-way:		0		330		
Actions:						
No communication:		325		5		
One-way:						
Rep. 1&3		88		132		
Rep. 2		15		95		
Total		103		227		
Two-way:		15		316		
		Acti	on pair			
	(1,1)	(2,2		(1,2), (2,1)		
Treatment:						
No communication:	160	0)	5		
One-way:						
Rep. 1&3	25	47	7	38		
Rep. 2	1	41	L	13		
Total	26	88	3	51		
Two-way:	0	150)	15		

Communication works, but particularly if it is 2-sided.

It appears to have a reassurance component in that both players can reassure each other of their intentions regarding each other.