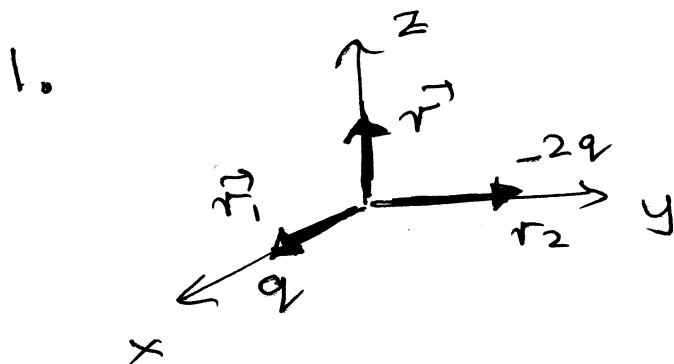


EM PROBLEM SET 4 2019

SOLUTIONS



The key formula is that for a charge q located at $\vec{r}_0$, the electric potential at a point $\vec{r}$ is

$$V(\vec{r}) = \frac{q}{4\pi\epsilon_0 |\vec{r} - \vec{r}_0|}$$

In the case here, $\vec{r} = (0, 0, c)$,

and $V(\vec{r}) = \frac{q}{4\pi\epsilon_0 |\vec{r} - \vec{r}_1|}$ ← electric potential due to q

$+ \frac{(-2q)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_2|}$ ← electric potential due to charge $-2q$

$$\vec{r} = (0, 0, c)$$

$$\vec{r}_1 = (a, 0, 0)$$

$$\vec{r} - \vec{r}_1 = (-a, 0, c)$$

$$|\vec{r} - \vec{r}_1| = \sqrt{(-a)^2 + 0 + c^2} = \sqrt{a^2 + c^2}$$

$$\vec{r}_1 = (0, 0, c)$$

$$\vec{r}_2 = (0, b, 0)$$

$$\vec{r}_1 - \vec{r}_2 = (0, -b, c)$$

$$|\vec{r}_1 - \vec{r}_2| = \sqrt{0 + (-b)^2 + c^2} = \sqrt{b^2 + c^2}$$

$$\begin{aligned} \text{So } V(\vec{r}) &= \frac{q}{4\pi\epsilon_0\sqrt{a^2+c^2}} \\ &\quad - \frac{2q}{4\pi\epsilon_0\sqrt{b^2+c^2}} \end{aligned}$$

$$2. \quad \vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$$

$$\begin{aligned} \vec{\nabla} V(\vec{r}) &= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) (\alpha z - \beta xy) \\ &= (-\beta y, -\beta x, \alpha) \end{aligned}$$

$$\begin{aligned} \Rightarrow \vec{E}(\vec{r}) &= (\beta y, \beta x, -\alpha) \\ &= \beta y \vec{e}_x + \beta x \vec{e}_y - \alpha \vec{e}_z \end{aligned}$$

$$3. \quad \vec{\nabla} \phi(\vec{r})$$

$$= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) (x^3 + y^3 + z^3)$$

$$= (3x^2, 3y^2, 3z^2)$$

$$= 3x^2 \vec{e}_x + 3y^2 \vec{e}_y + 3z^2 \vec{e}_z$$

$$4. \quad \vec{\nabla} \cdot \vec{A}(\vec{r})$$

$$= (\partial_x, \partial_y, \partial_z) \cdot (A_x(\vec{r}), A_y(\vec{r}), A_z(\vec{r}))$$

$$= \partial_x A_x(\vec{r}) + \partial_y A_y(\vec{r}) + \partial_z A_z(\vec{r})$$

$$= \frac{\partial}{\partial x} (xz) + \frac{\partial}{\partial y} (z) + \frac{\partial}{\partial z} (-xy)$$

$$= z + 0 + 0$$

$$= z$$

$$5. \quad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\vec{A}(\vec{r}) = (xz, z, -xy)$$

Method 1:

$$\begin{aligned} (\vec{\nabla} \times \vec{A}(\vec{r}))_x &= \frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \\ &= \frac{\partial}{\partial y} (-xy) - \frac{\partial}{\partial z} (z) \\ &= -x - 1 \end{aligned}$$

$$\begin{aligned}
 (\vec{\nabla} \times \vec{A}(\vec{r}))_y &= \frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \\
 &= \frac{\partial}{\partial z} (xz) - \frac{\partial}{\partial x} (-xy) \\
 &= x - (-y) \\
 &= x + y
 \end{aligned}$$

$$\begin{aligned}
 (\vec{\nabla} \times \vec{A}(\vec{r}))_z &= \frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \\
 &= \frac{\partial}{\partial x} (z) - \frac{\partial}{\partial y} (xz) \\
 &= 0 - 0 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{So } \vec{\nabla} \times \vec{A}(\vec{r}) &= -(x+1) \vec{e}_x \\
 &\quad + (x+y) \vec{e}_y
 \end{aligned}$$

Method 2 :

$$\begin{array}{c}
 \vec{\nabla} : \begin{array}{c} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{array} \quad \begin{array}{c} \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \quad \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \end{array} \quad \begin{array}{c} \frac{\partial}{\partial z} \\ -xy \end{array} \\
 \vec{A} : \begin{array}{c} xz \\ z \\ -xy \\ xz \\ z \end{array}
 \end{array}$$

$(\vec{\nabla} \times \vec{A})_x = \frac{\partial}{\partial y} (-xy) - \frac{\partial}{\partial z} (z) = -x - 1$

$(\vec{\nabla} \times \vec{A})_y = \frac{\partial}{\partial z} (xz) - \frac{\partial}{\partial x} (-xy) = x - (-y) = x + y$

$(\vec{\nabla} \times \vec{A})_z = \frac{\partial}{\partial x} (z) - \frac{\partial}{\partial y} (xz) = 0$