HEAT TRANSFER

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One-Dimensional, Steady-State Conduction

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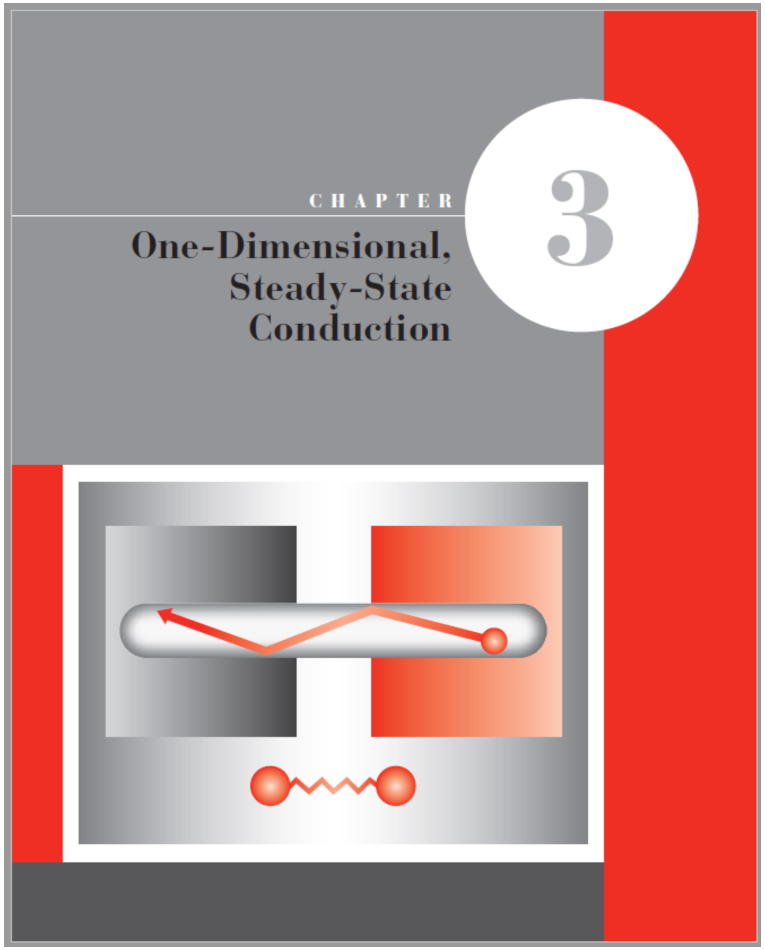


One-Dimensional, Steady-State Conduction

- Conduction problems may involve multiple directions and time-dependent conditions
- Inherently complex-difficult to determine temperature distributions.
- In a one-dimensional system, temperature gradients exist along only a single coordinate direction, and heat transfer (diffusion) occurs exclusively in that direction.
- In steady-state conditions, the temperature at each point is independent of time.
- First, the heat transfer under this conditions will be discussed considering that there is no heat generation.
- The heat resistance will be introduced.

One-Dimensional, Steady-State Conduction

Methodology of a conduction analysis

1. Specify appropriate form of the heat equation
2. Solve for the temperature distribution
3. Apply Fourier's law to determine the heat flux

Simplest case:

One-dimensional, steady state conduction with no thermal energy generation

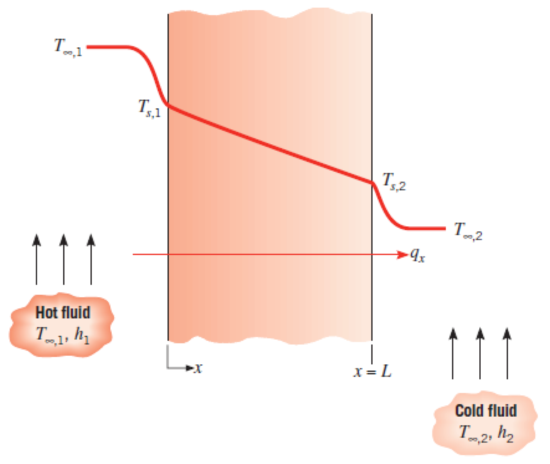
Common geometries:

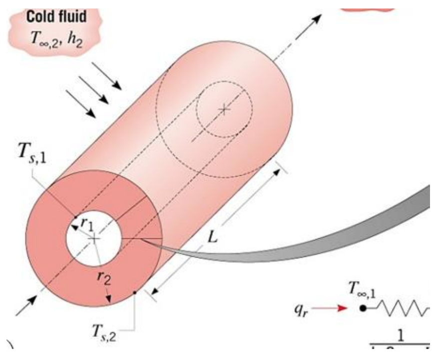
- ❖ The plane wall: described in rectangular (x) coordinate. Area perpendicular to direction of heat transfer is constant (independent of x).
- ❖ Cylindrical wall: radial conduction through tube wall.
- ❖ Spherical wall: radial conduction through shell wall.

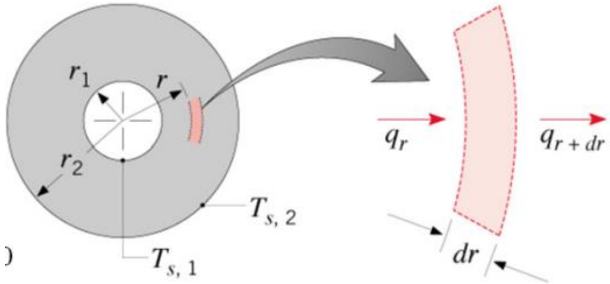
One-Dimensional, Steady-State Conduction

Methodology of a conduction analysis

Common geometries



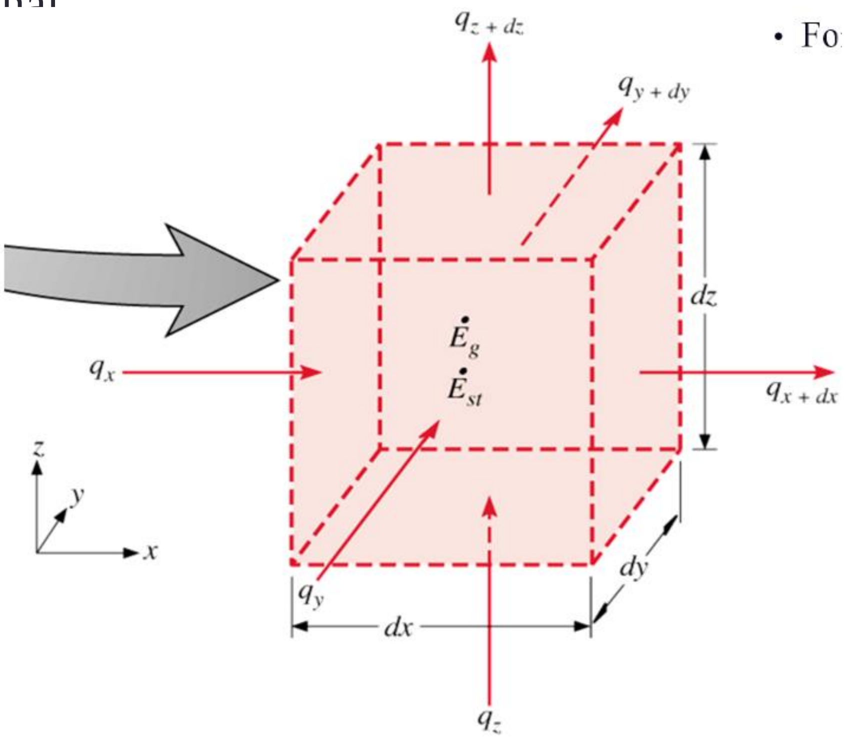




One-Dimensional, Steady-State Conduction

1. The Plane Wall

Case e of 3-dimension



• Fo

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

One-Dimensional, Steady-State Conduction

1. The Plane Wall

Using the following equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

Considering: (I) no heat generation, (II) steady-state conditions and (Ii) one-dimensional flow:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$$

For constant k and A, this equation will be a second order differential equation:

$$\frac{d^2T}{dx^2}=0$$

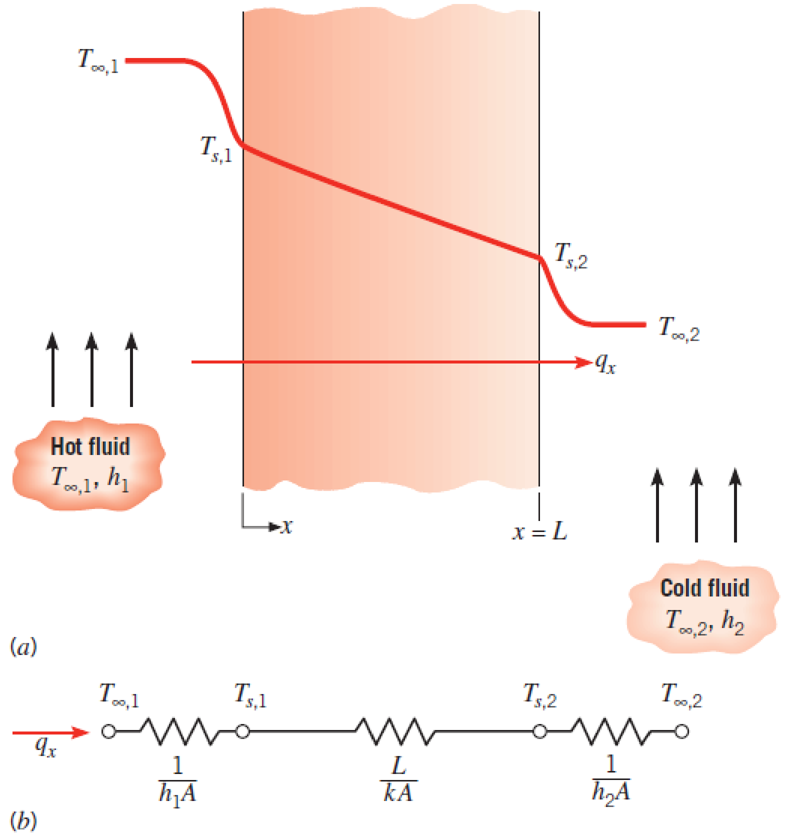


FIGURE 3.1 Heat transfer through a plane wall. (*a*) Temperature distribution. (*b*) Equivalent thermal circuit.

One-Dimensional, Steady-State Conduction

1. The Plane Wall

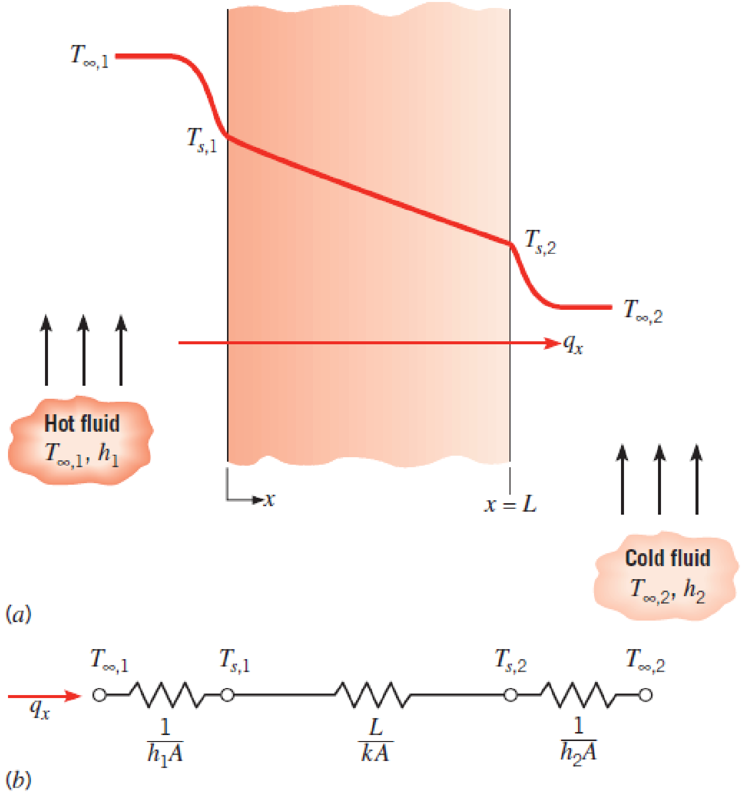
1-D heat conduction equation for steady-state conditions and no internal heat generation (i.e. q = 0), is

$$\frac{d^2T}{dx^2}=0$$ ***for constant k and A***

This mean:

Heat flux (q''_x) is independent of x

Heat rate (q_x) is independent of x



The general solution of the equation is:

$$T(x)=\int\int\frac{d^2T}{dx^2}\,dx\,dx=\int(\frac{dT}{dx}+C_1)\,dx$$

$$\therefore T(x)=C_1\cdot x+C_2$$

Where C1 and C1, are constants which can be found from the ***<u>boundary conditions</u>*** of the system.

One-Dimensional, Steady-State Conduction

1. The Plane Wall

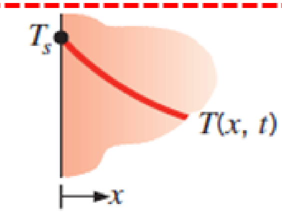
Boundary conditions

In this system, first kind boundaries were used

1. Constant surface temperature

Ice bath or heat sink

$$T(0, t) = T_s \qquad (2.31)$$



For boundary conditions: $T(0) = T_{s,1}$ and $T(L) = T_{s,2}$

at x = 0, T(x) = $T_{s,1}$ and $C_2 = T_{s,1}$

at x = L, T(x) = $T_{s,2}$ and $T_{s,2} = C_1 L + C_2 = C_1 L + T_{s,1}$

this gives, $C_1 = (T_{s,2} - T_{s,1})/2$

Using value of C_1 and C_2, the function of $T(x)$ is

$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1} \Rightarrow \frac{dT}{dx} = \frac{T_{s,2} - T_{s,1}}{L}$$

From here, apply Fourier's law to get heat transfer.

One-Dimensional, Steady-State Conduction

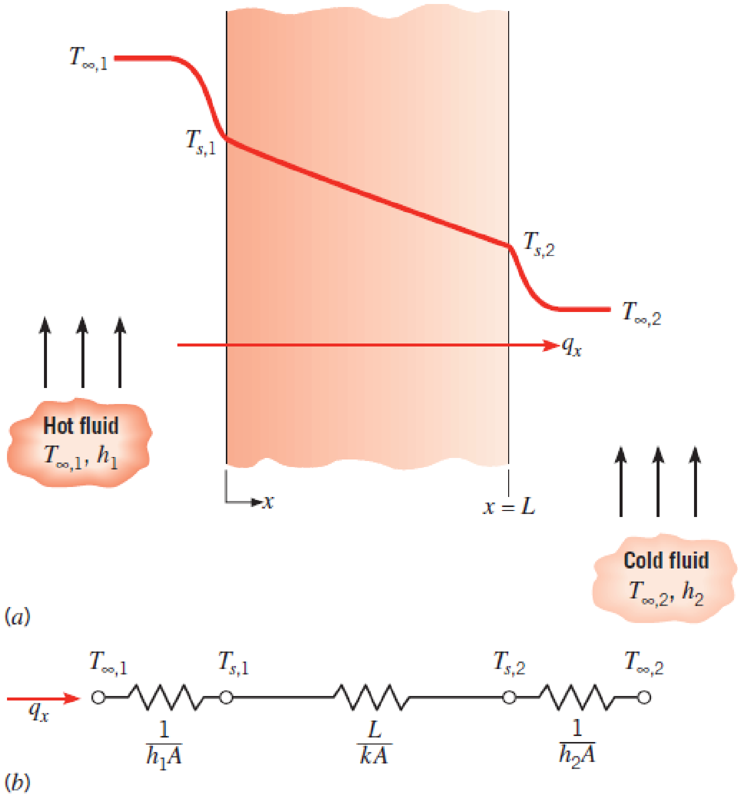
1. The Plane Wall

Heat flux for plane wall:

$$q''_x = -k\frac{dT}{dx} = \frac{k}{L}\left(T_{s,1} - T_{s,2}\right)$$

Heat rate for plane wall:

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right)$$



One-Dimensional, Steady-State Conduction

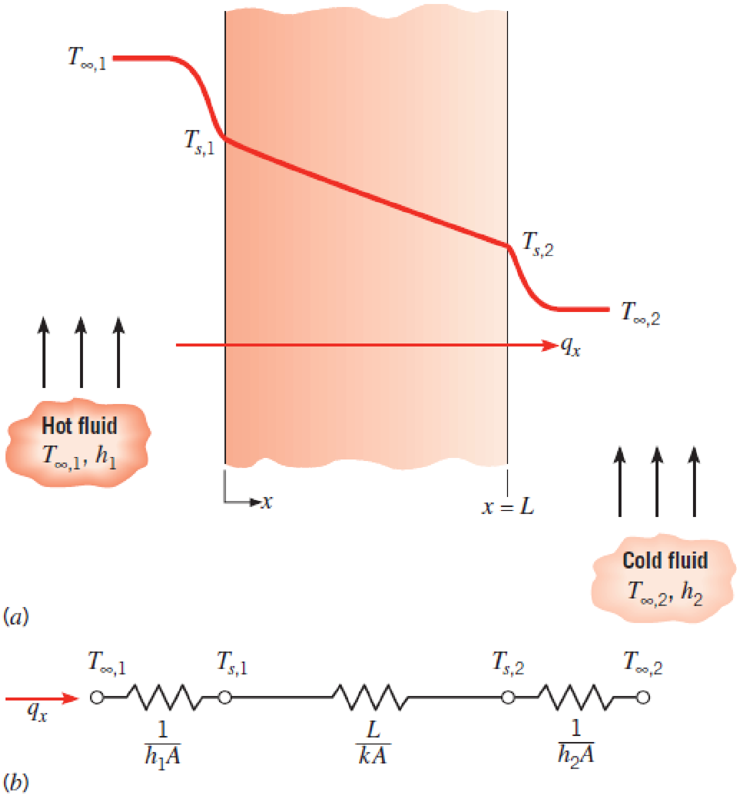
1. The Plane Wall

Heat flux for plane wall:

$$q''_x = \frac{q_x}{A} = \frac{k}{L}(T_{s,1} - T_{s,2})$$

Heat rate for plane wall:

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}(T_{s,1} - T_{s,2})$$



One-Dimensional, Steady-State Conduction

2. Thermal Resistance

Just like the electrical conductivity and electrical resistivity, the thermal conductivity and thermal resistivity are related to each other:

- **Recall electric circuit theory - Ohm's law for electrical resistance:**

$$\text{Electric current} = \frac{\text{Potential Difference}}{\text{Resistance}}$$

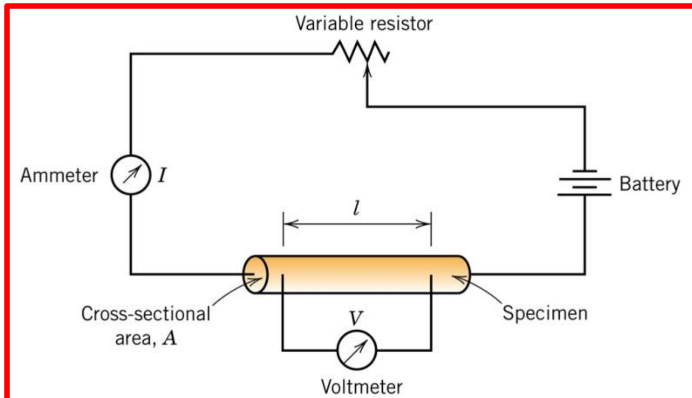
- By the same way, the heat transfer rate equation can be written as:

$$-kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) = \frac{\left(T_{s,1} - T_{s,2}\right)}{L/kA}$$

One-Dimensional, Steady-State Conduction

2. Thermal Resistance

Just like the electrical conductivity and electrical resistivity, the thermal conductivity and thermal resistivity are related to each other:



$$\rho = \frac{RA}{l}$$

surface area of current flow

current flow path length

$$\sigma = \frac{1}{\rho}$$

$$R_e = \frac{L}{\sigma A}$$

$$R_{t,\mathrm{cond}} = \frac{L}{kA} \equiv \frac{T_{s,1} - T_{s,2}}{q_x}$$

One-Dimensional, Steady-State Conduction

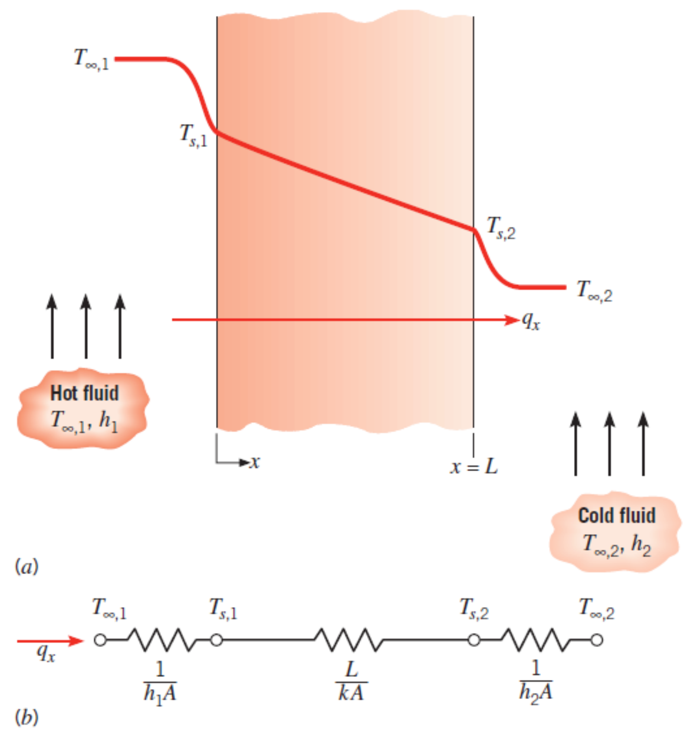
2. Thermal Resistance

Considering the convection heat transfer at the surface:

$$q = hA(T_s - T_\infty) \quad \Rightarrow \quad \boxed{R_{t,\text{conv}} \equiv \frac{T_s - T_\infty}{q} = \frac{1}{hA}}$$

- In this regard, the modelling of heat resistance using circuit representations will be needed to include both modes.
- The equivalent thermal circuit for the plane wall with convection surface conditions is shown here:
- Since q_x is constant throughout the system:

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$



One-Dimensional, Steady-State Conduction

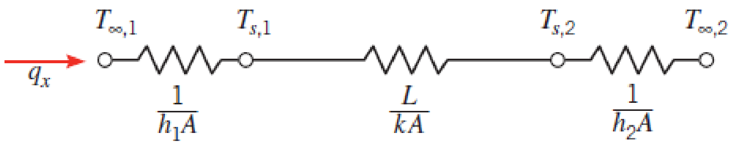
2. Thermal Resistance

We can use this electrical analogy to represent heat transfer problems using the concept of *a thermal circuit* (equivalent to an electrical circuit).

$$q_x = \frac{\text{Overall Driving Force}}{\text{Resistance}} = \frac{\Delta T_{overall}}{\sum R}$$

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A} \Rightarrow q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{\text{tot}}}$$

The conduction and convection resistances are in series and can be summed



$$R_{\text{tot}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

One-Dimensional, Steady-State Conduction

3. The Composite Wall

The thermal circuits for more complex systems, such as composite walls, in which various layers can be included in the wall.

- For the wall shown here, the following equation can be used to introduce the heat transfer rate:

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\Sigma R_t}$$

where $T_{\infty,1}$-$T_{\infty,4}$ is the overall temperature difference.

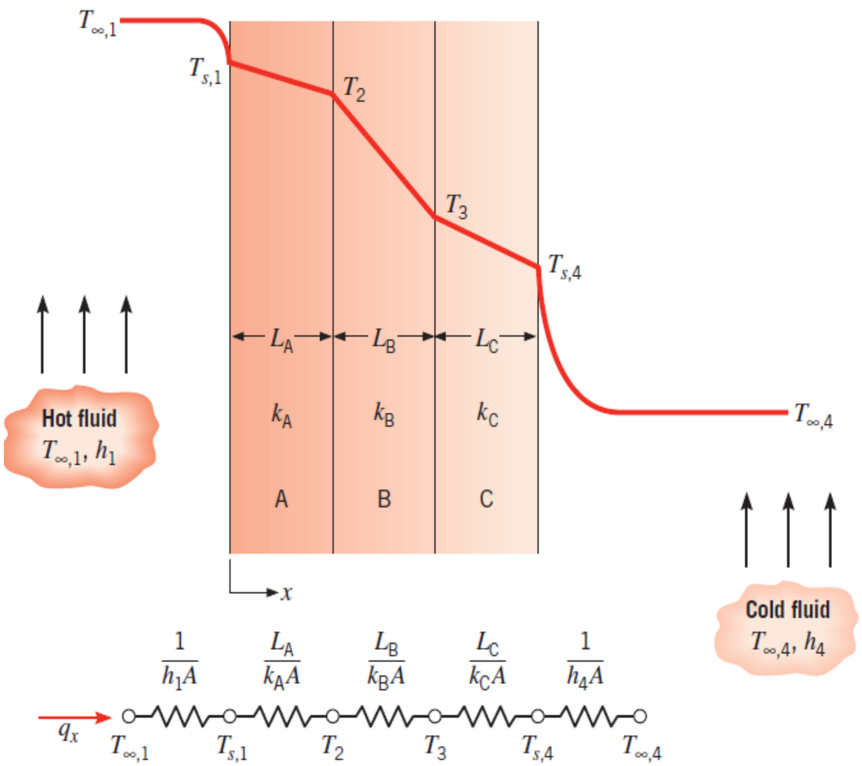


FIGURE 3.2 Equivalent thermal circuit for a series composite wall.

One-Dimensional, Steady-State Conduction

3. The Composite Wall

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{[(1/h_1 A) + (L_A/k_A A) + (L_B/k_B A) + (L_C/k_C A) + (1/h_4 A)]}$$

For this wall, the overall heat transfer coefficient U is used:

$$U = \frac{1}{R_{tot} A} = \frac{1}{[(1/h_1) + (L_A/k_A) + (L_B/k_B) + (L_C/k_C) + (1/h_4)]}$$

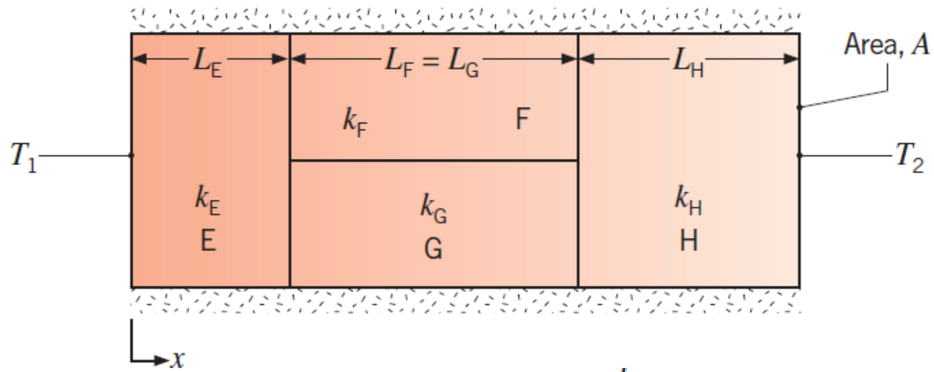
$$q_x \equiv UA\,\Delta T$$

$$R_{tot} = \sum R_t = \frac{\Delta T}{q} = \frac{1}{UA}$$

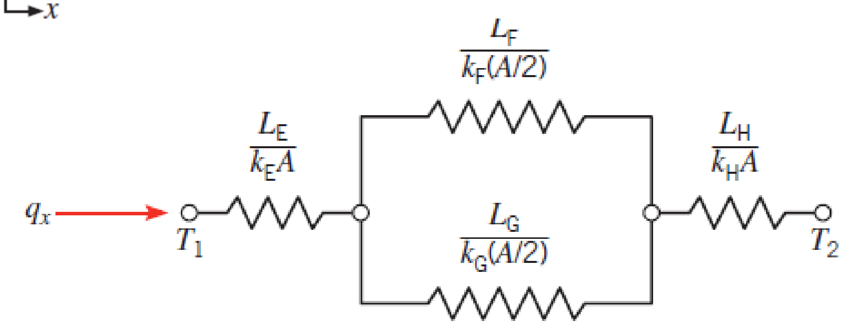
One-Dimensional, Steady-State Conduction

3. The Composite Wall

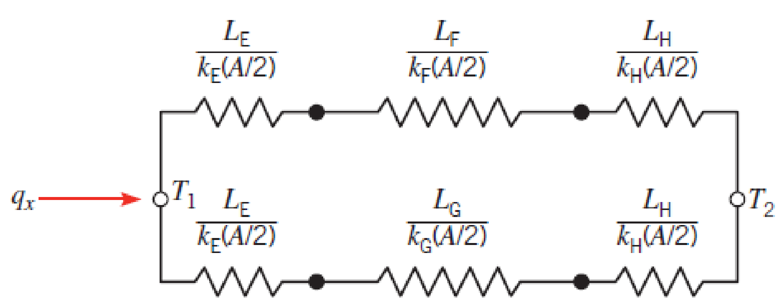
Series-parallel type



1. Surfaces normal to the x-direction are isothermal



2. Surfaces parallel to the x-direction are adiabatic



One-Dimensional, Steady-State Conduction

4. Contact Resistance

- Although neglected until now, it is important to recognize that, in composite systems, the temperature drop across the interface between materials may be appreciable.
- This temperature change is attributed to what is known as the thermal contact resistance, $R_{t,c}$.

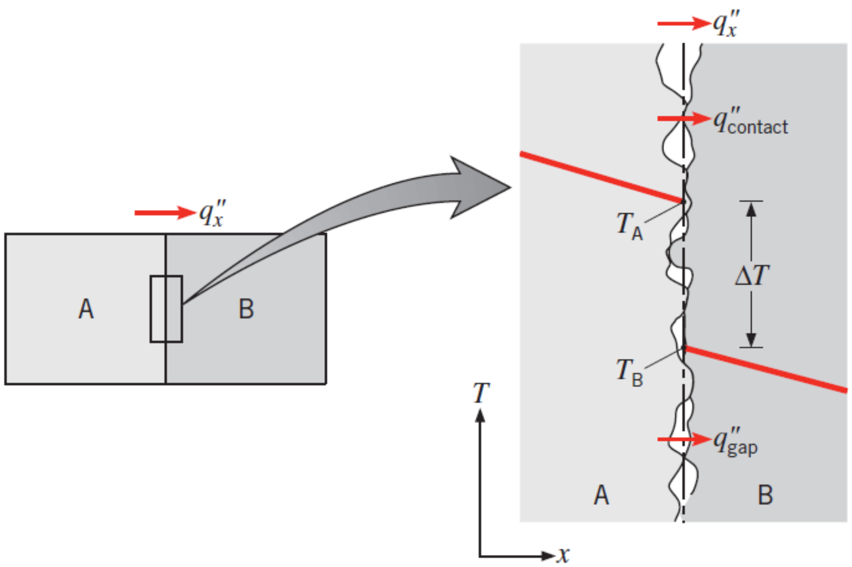


FIGURE 3.4 Temperature drop due to thermal contact resistance.

$$R''_{t,c} = \frac{T_A - T_B}{q''_x}$$

$$q''_x = q''_{gap} + q''_{cont}$$

One-Dimensional, Steady-State Conduction

4. Contact Resistance

TABLE 3.1 Thermal contact resistance for (*a*) metallic interfaces under vacuum conditions and (*b*) aluminum interface (10-μm surface roughness, 10^5 N/m^2) with different interfacial fluids [1]

Thermal Resistance, $R''_{t,c} \times 10^4$ ($m^2 \cdot K/W$)

(*a*) Vacuum Interface			(*b*) Interfacial Fluid	
Contact pressure	100 kN/m^2	10,000 kN/m^2		
Stainless steel	6–25	0.7–4.0	Air	2.75
Copper	1–10	0.1–0.5	Helium	1.05
Magnesium	1.5–3.5	0.2–0.4	Hydrogen	0.720
Aluminum	1.5–5.0	0.2–0.4	Silicone oil	0.525
			Glycerine	0.265

One-Dimensional, Steady-State Conduction

4. Contact Resistance

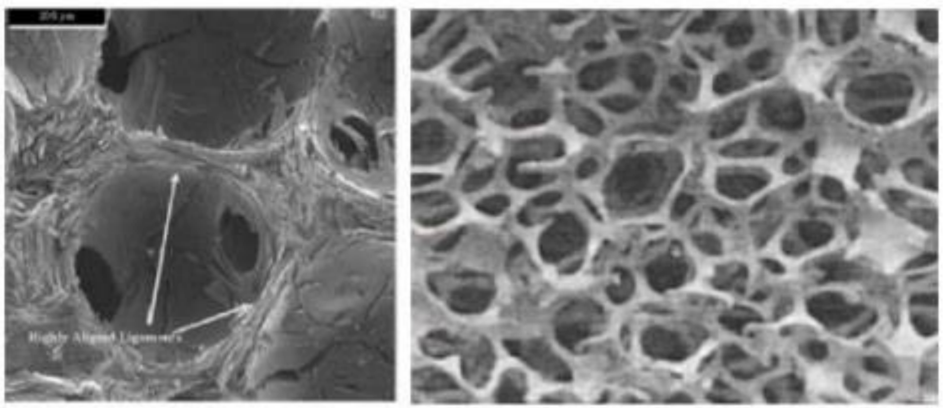
TABLE 3.2 Thermal resistance of representative solid/solid interfaces

Interface	$R''_{t,c} \times 10^4$ $(m^2 \cdot K/W)$	Source
Silicon chip/lapped aluminum in air (27–500 kN/m^2)	0.3–0.6	[2]
Aluminum/aluminum with indium foil filler (~100 kN/m^2)	~0.07	[1, 3]
Stainless/stainless with indium foil filler (~3500 kN/m^2)	~0.04	[1, 3]
Aluminum/aluminum with metallic (Pb) coating	0.01–0.1	[4]
Aluminum/aluminum with Dow Corning 340 grease (~100 kN/m^2)	~0.07	[1, 3]
Stainless/stainless with Dow Corning 340 grease (~3500 kN/m^2)	~0.04	[1, 3]
Silicon chip/aluminum with 0.02-mm epoxy	0.2–0.9	[5]
Brass/brass with 15-μm tin solder	0.025–0.14	[6]

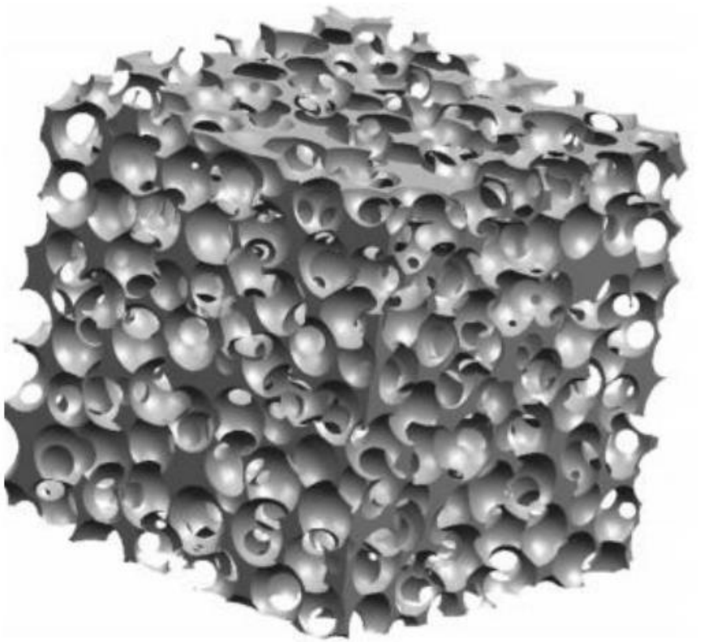
One-Dimensional, Steady-State Conduction

5. Porous Media

In many applications, heat transfer occurs within *porous media* that are combinations of a stationary solid and a fluid.



Photographs of (a) Scanning Electron Microscope (SEM) image showing the cross section of carbon foam, and (b) an aluminum foam consisting of interconnected ligaments



One-Dimensional, Steady-State Conduction

5. Porous Media

Consider a saturated porous medium that is subjected to surface temperatures T_1 at $x = 0$ and T_2 at $x = L$. After steady-state conditions are reached and if T_1 T_2, the heat rate may be expressed as:

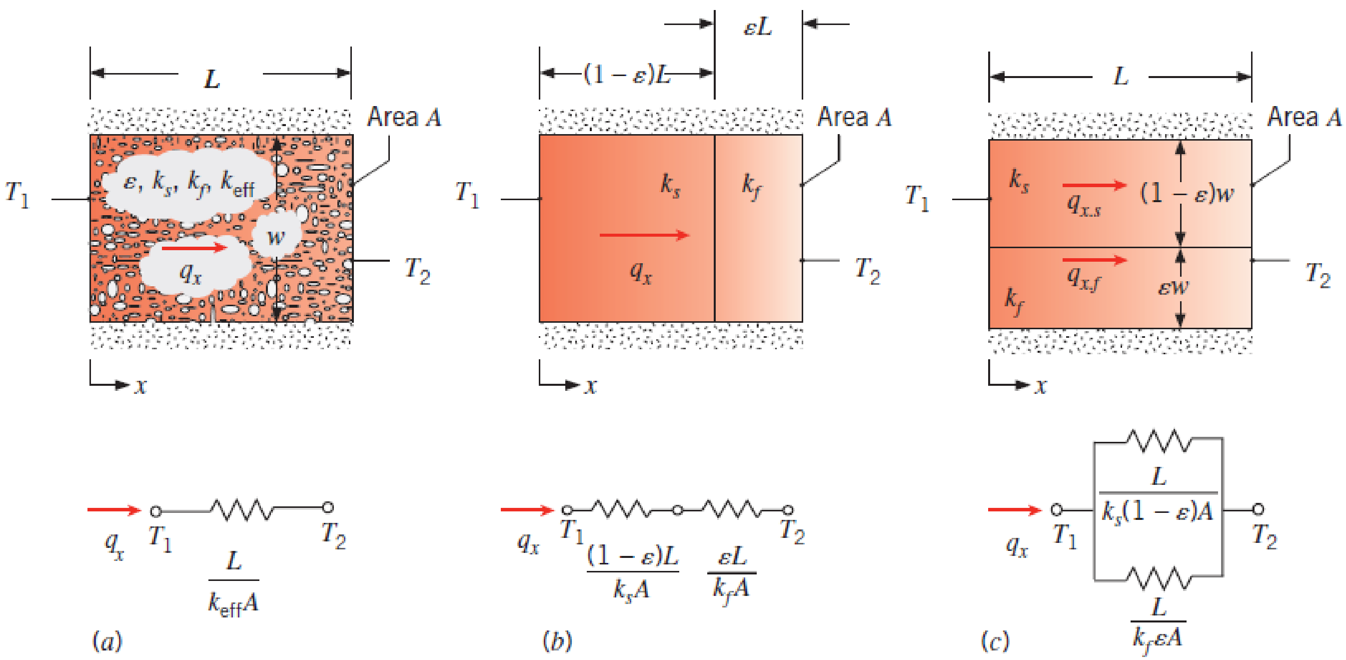


FIGURE 3.5 A porous medium. (*a*) The medium and its properties. (*b*) Series thermal resistance representation. (*c*) Parallel resistance representation.

One-Dimensional, Steady-State Conduction

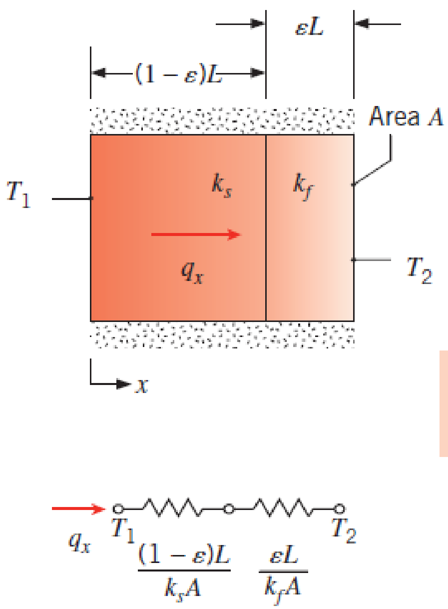
5. Porous Media

Heat rate may be expressed as

$$q_x = \frac{k_{\text{eff}} A}{L}(T_1 - T_2)$$

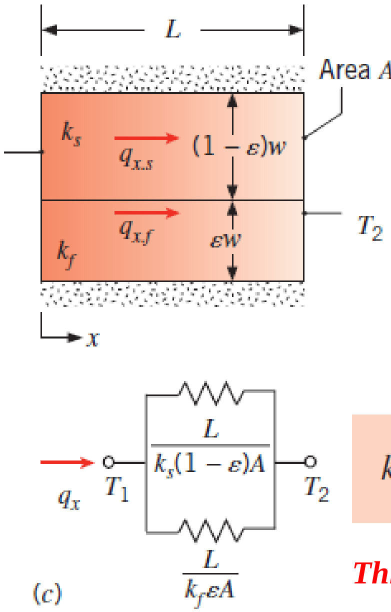
where k_{eff} is an effective thermal conductivity of the porous system.

- This equation is valid when there is no radiation heat transfer.



$$q_x = \frac{A\,\Delta T}{(1-\varepsilon)L/k_s + \varepsilon L/k_f}$$

$$k_{\text{eff,min}} = \frac{1}{(1-\varepsilon)/k_s + \varepsilon/k_f}$$



$$k_{\text{eff,max}} = \varepsilon k_f + (1-\varepsilon)k_s$$

$$k_{\text{eff}} = \left[\frac{k_f + 2k_s - 2\varepsilon(k_s - k_f)}{k_f + 2k_s + \varepsilon(k_s - k_f)}\right]k_s$$

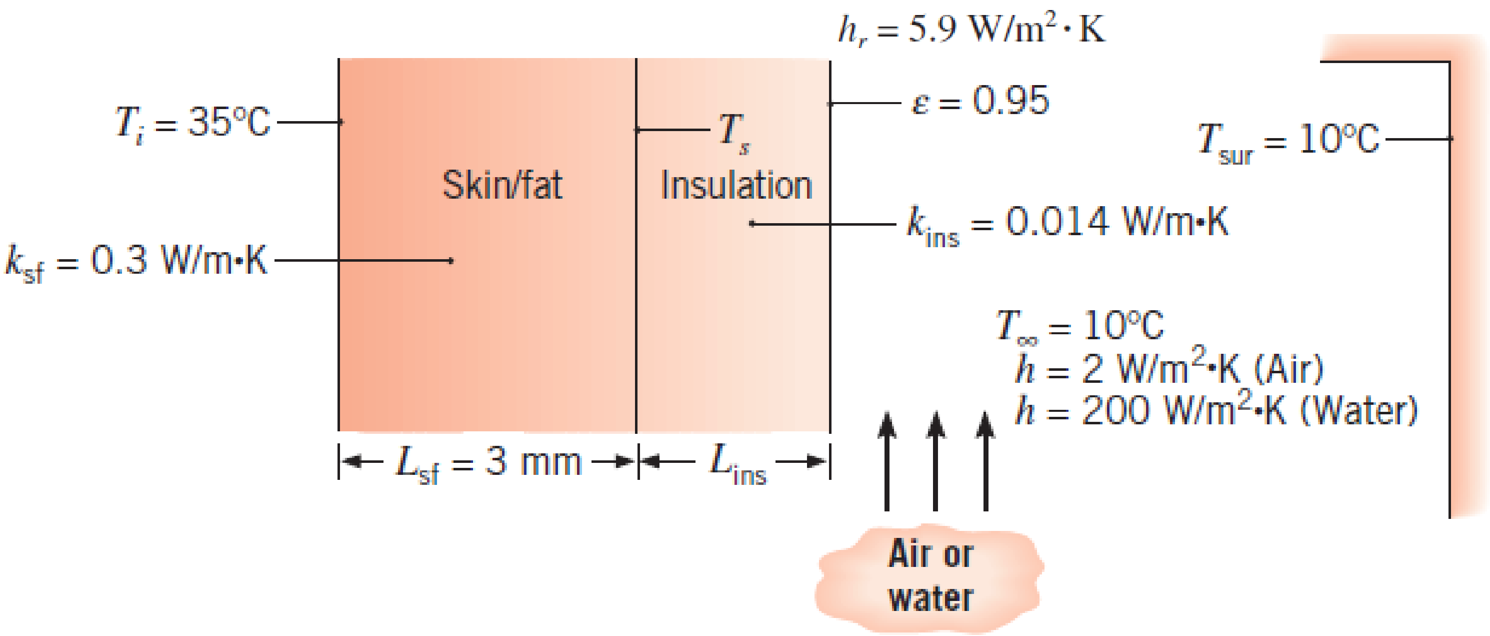
This is valid for porosity less than 25%

One-Dimensional, Steady-State Conduction

1. The Plane Wall

Example-1

- Heat loss from the skin surface is (I) 146 W (air) and (II) 1320 W (water).
- Calculate the thickness of the insulator needed to reduce the heat transfer rate from this skin to 100 W in both cases (air and water).

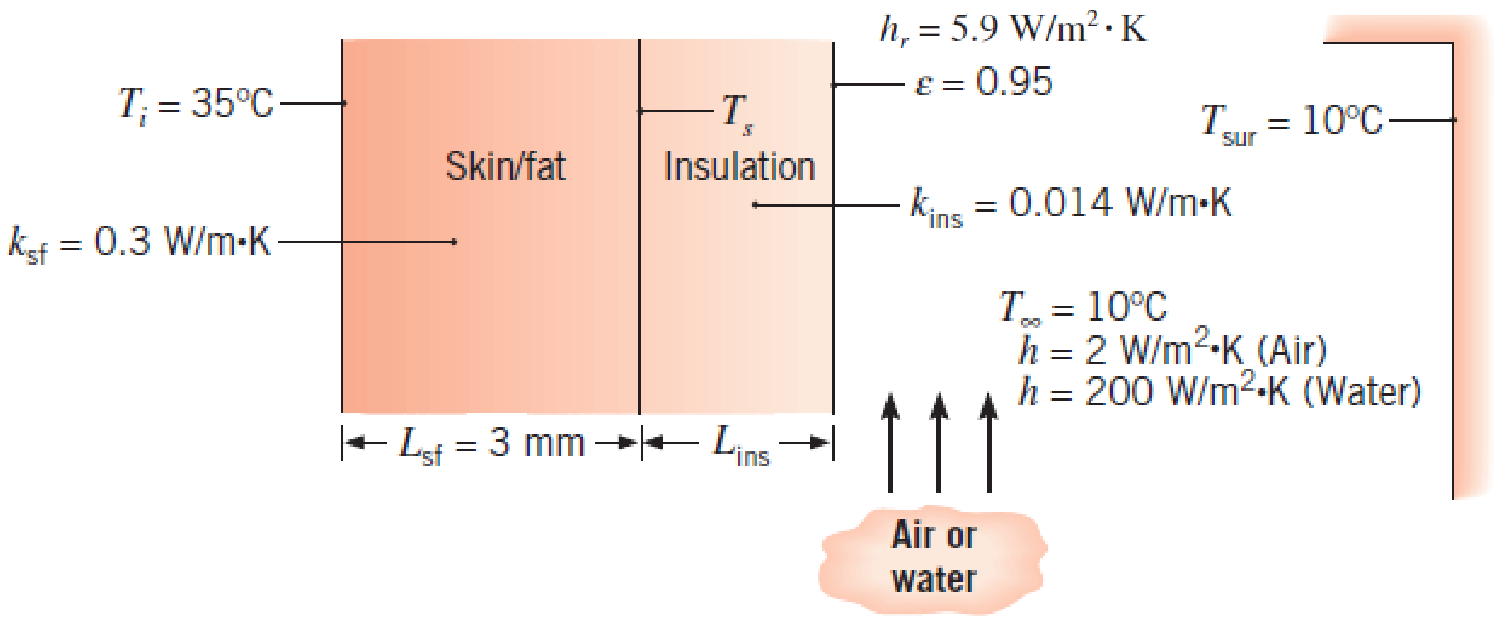


One-Dimensional, Steady-State Conduction

1. The Plane Wall

Example-1

- To solve this problem, first the heat transfer modes, by which the heat from the skin is lost, should be introduced.
- Total resistance concept should be used.



One-Dimensional, Steady-State Conduction

1. The Plane Wall

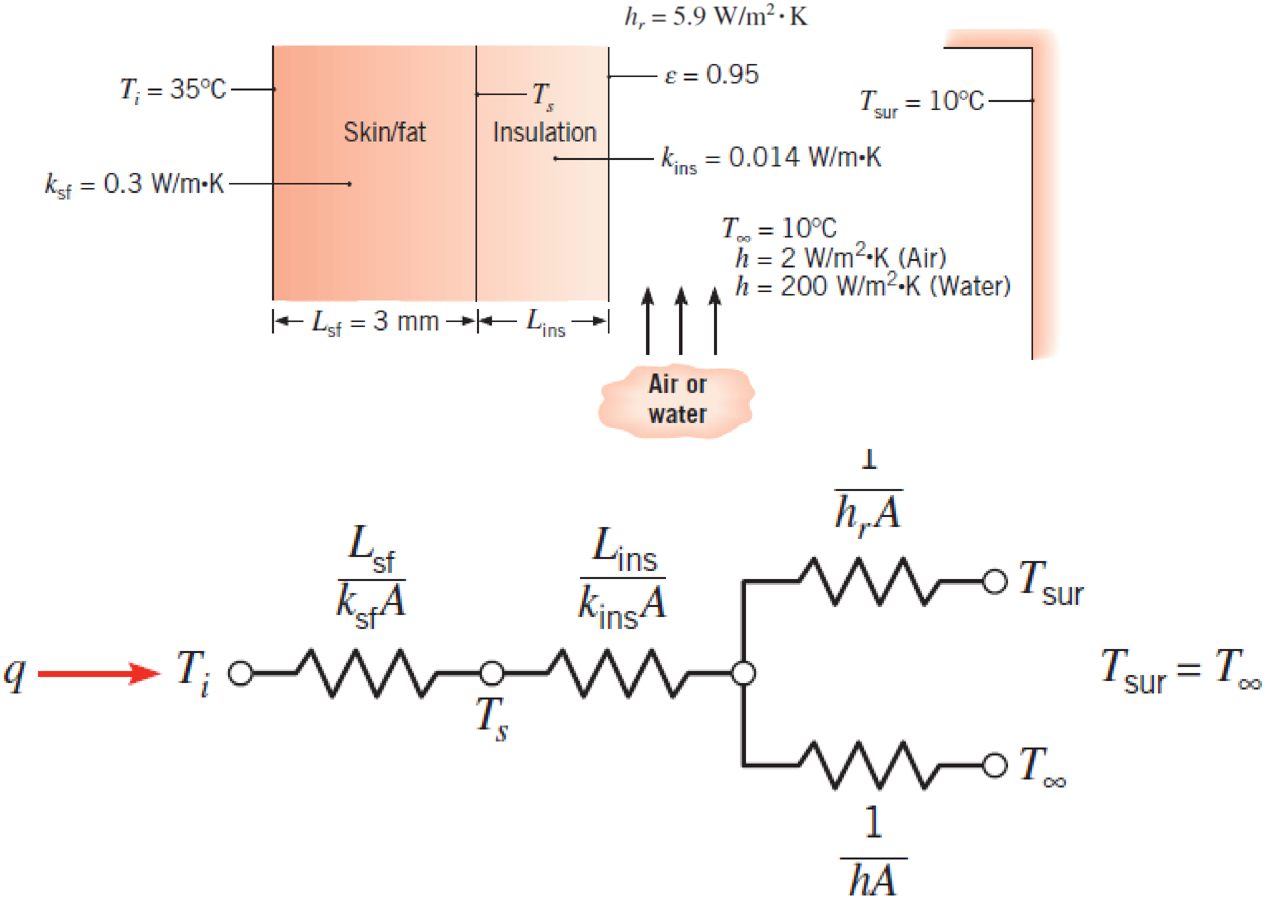
Example-1

- Steady-state conditions.
- One-dimensional heat transfer by conduction through the skin/fat and insulation layers.
- Contact resistance is negligible.
- Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
- Liquid water is opaque to thermal radiation.

One-Dimensional, Steady-State Conduction

1. The Plane Wall

Example-1



One-Dimensional, Steady-State Conduction

1. The Plane Wall

Example-1

$$R_{\text{tot}} = \frac{T_i - T_\infty}{q} = \frac{(35 - 10)\text{ K}}{100\text{ W}} = 0.25\text{ K/W}$$

$$R_{\text{tot}} = \frac{L_{\text{sf}}}{k_{\text{sf}}A} + \frac{L_{\text{ins}}}{k_{\text{ins}}A} + \left(\frac{1}{1/hA} + \frac{1}{1/h_r A}\right)^{-1} = \frac{1}{A}\left(\frac{L_{\text{sf}}}{k_{\text{sf}}} + \frac{L_{\text{ins}}}{k_{\text{ins}}} + \frac{1}{h + h_r}\right)$$

<u>Air</u>

$$L_{\text{ins}} = k_{\text{ins}}\left[AR_{\text{tot}} - \frac{L_{\text{sf}}}{k_{\text{sf}}} - \frac{1}{h + h_r}\right]$$

$$= 0.014\text{ W/m}\cdot\text{K}\left[1.8\text{ m}^2 \times 0.25\text{ K/W} - \frac{3 \times 10^{-3}\text{ m}}{0.3\text{ W/m}\cdot\text{K}} - \frac{1}{(2 + 5.9)\text{ W/m}^2\cdot\text{K}}\right]$$

$$= 0.0044\text{ m} = 4.4\text{ mm}$$

One-Dimensional, Steady-State Conduction

1. The Plane Wall

Example-1

Water

$$R_{\text{tot}} = \frac{L_{\text{sf}}}{k_{\text{sf}}A} + \frac{L_{\text{ins}}}{k_{\text{ins}}A} + \left(\frac{1}{1/hA} + \frac{1}{1/h_r A}\right)^{-1} = \frac{1}{A}\left(\frac{L_{\text{sf}}}{k_{\text{sf}}} + \frac{L_{\text{ins}}}{k_{\text{ins}}} + \frac{1}{h + h_r}\right)$$

$$L_{\text{ins}} = k_{\text{ins}}\left[AR_{\text{tot}} - \frac{L_{\text{sf}}}{k_{\text{sf}}} - \frac{1}{h}\right]$$

$$= 0.014\ \text{W/m}\cdot\text{K}\left[1.8\ \text{m}^2 \times 0.25\ \text{K/W} - \frac{3\times 10^{-3}\ \text{m}}{0.3\ \text{W/m}\cdot\text{K}} - \frac{1}{200\ \text{W/m}^2\cdot\text{K}}\right]$$

$$= 0.0061\ \text{m} = 6.1\ \text{mm}$$

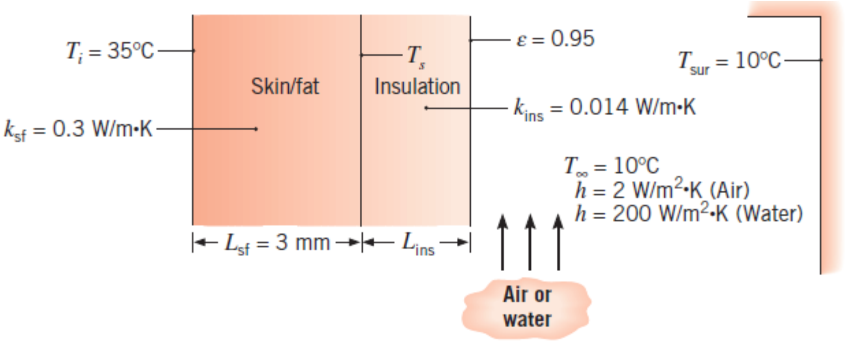
One-Dimensional, Steady-State Conduction

1. The Plane Wall

Example-1

The skin surface temperature can be calculated by considering conduction through the skin/fat layer:

$$q = \frac{k_{sf} A (T_i - T_s)}{L_{sf}}$$



$$T_s = T_i - \frac{qL_{sf}}{k_{sf}A} = 35°\text{C} - \frac{100\ \text{W} \times 3 \times 10^{-3}\ \text{m}}{0.3\ \text{W/m} \cdot \text{K} \times 1.8\ \text{m}^2} = 34.4°\text{C}$$