## CHAPTER 6 <br> DISCOUNTED CASH FLOW VALUATION

## Solutions to Questions and Problems

2. The time lines are:


To find the PVA, we use the equation:
PVA $=C\left(\left\{1-\left[1 /(1+r)^{\dagger}\right]\right\} / r\right)$
At a 5 percent interest rate:
X@5\%: PVA $=\$ 4,200\left\{\left[1-(1 / 1.05)^{8}\right] / .05\right\}=\$ 27,145.49$
$\mathrm{Y} @ 5 \%: \quad \mathrm{PVA}=\$ 6,100\left\{\left[1-(1 / 1.05)^{5}\right] / .05\right\}=\$ 26,409.81$
And at a 15 percent interest rate:
X@15\%: PVA $=\$ 4,200\left\{\left[1-(1 / 1.15)^{8}\right] / .15\right\}=\$ 18,846.75$
$\mathrm{Y} @ 15 \%: \mathrm{PVA}=\$ 6,100\left\{\left[1-(1 / 1.15)^{5}\right] / .15\right\}=\$ 20,448.15$
Notice that the PV of Cash flow X has a greater PV at a 5 percent interest rate, but a lower PV at a 15 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At the higher interest rate, these larger cash flows early are more important since the cost of waiting (the interest rate) is so much greater.
5. The time line is:


Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

$$
\begin{aligned}
& \text { PVA }=C(\{1-[1 /(1+r)\rceil\} / r) \\
& \text { PVA }=\$ 41,000=\$ C\left\{\left[1-\left(1 / 1.051^{15}\right)\right] / .051\right\}
\end{aligned}
$$

We can now solve this equation for the annuity payment. Doing so, we get:
C $=\$ 41,000 / 10.30985$
$C=\$ 3,976.78$
10. The time line is:


This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:
$\mathrm{PV}=C / r$
$\mathrm{PV}=\$ 35,000 / .047$
$\mathrm{PV}=\$ 744,680.85$
12. For discrete compounding, to find the EAR, we use the equation:
$\mathrm{EAR}=[1+(\mathrm{APR} / m)]^{m}-1$
$\operatorname{EAR}=[1+(.09 / 4)]^{4}-1=.0931$, or $9.31 \%$
$\operatorname{EAR}=[1+(.16 / 12)]^{12}-1=.1723$, or $17.23 \%$
$\mathrm{EAR}=[1+(.12 / 365)]^{365}-1=.1275$, or $12.75 \%$
To find the EAR with continuous compounding, we use the equation:
EAR $=\mathrm{e}^{q}-1$
$\operatorname{EAR}=\mathrm{e}^{.11}-1$
$\mathrm{EAR}=.1163$, or $11.63 \%$
20. The time line is:


We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:
$\mathrm{PVA}=C\left(\left\{1-\left[1 /(1+r)^{\mathrm{t}}\right]\right\} / r\right)$
$\$ 84,500=\$ C\left[1-\left\{1 /[1+(.052 / 12)]^{60}\right\} /(.052 / 12)\right]$
Solving for the payment, we get:
$C=\$ 84,500 / 52.7343$
$C=\$ 1,602.37$
To find the EAR, we use the EAR equation:
$\mathrm{EAR}=[1+(\mathrm{APR} / m)]^{m}-1$
$\mathrm{EAR}=[1+(.052 / 12)]^{12}-1$
$\mathrm{EAR}=.0533$, or $5.33 \%$

