Lecture 2

What Do Interest Rates Mean
and What Is Their Role in Valuation?

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Today's Topics

- How to measure interest rates?
- The distinction between real interest rate and nominal interest rate
- The distinction between interest rates and returns
- Why interest rate changes?

Present Value Introduction

- Different debt instruments have very different streams of cash payments to the holder (known as **cash flows**), with very different timing.
- All else being equal, debt instruments are evaluated against one another based on the *amount* of each cash flow and the *timing* of each cash flow.
- This evaluation, where the analysis of the amount and timing of a debt instrument's cash flows lead to its **yield to maturity (YTM)** or interest rate, is called *present value* analysis.

Present Value

- The concept of **present value** (or **present discounted value**) is based on the commonsense notion of **time value** of money.

- The term present value (PV) can be extended to mean the PV of a single cash flow or the ***sum*** of a sequence or group of cash flows.

Present Value Applications

- There are four basic types of credit instruments which incorporate present value concepts:
  1. Simple Loan
  2. Fixed Payment Loan
  3. Coupon Bond
  4. Discount Bond

Present Value Concept: Simple Loan Terms

- ***Loan Principal**:* the amount of funds the lender provides to the borrower.
- ***Maturity*** *Date:* the date the loan must be repaid; the *Loan Term* is from initiation to maturity date.
- ***Interest Payment**:* the cash amount that the borrower must pay the lender for the use of the loan principal.
- ***Simple Interest Rate**:* the interest payment divided by the loan principal; the percentage of principal that must be paid as interest to the lender. Convention is to express on an annual basis, irrespective of the loan term.

Present Value Concept: Simple Loan

Simple loan of $100

Year:	0	1	2	3	n
	$100	$110	$121	$133	$100\times(1+i)^n$

$$\text{PV of future \$1} = \frac{\$1}{(1+i)^n}$$

Present Value Concept: Simple Loan (cont.)

- The previous example reinforces the concept that $100 today is preferable to $100 a year from now since today's $100 could be lent out (or deposited) at 10% interest to be worth $110 one year from now, or $121 in two years or $133 in three years.

YTM Calculation – Simple Loan

- YTM = interest rate that equates today's value with present value of all future payments

1. Simple Loan Interest Rate (i = 10%)

$$\$100 = \$110/(1+i) \Rightarrow$$

$$i = \frac{\$110 - \$100}{\$100} = \frac{\$10}{\$100} = .10 = 10\%$$

Present Value of Cash Flows: Example

example 3.1 Simple Present Value

What is the present value of $250 to be paid in two years if the interest rate is 15%?

Solution

The present value would be $189.04. Using Equation 1:

$$PV = \frac{CF}{(1 + i)^n}$$

where

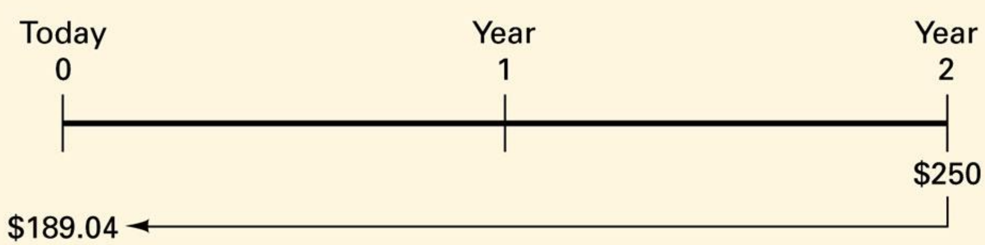
CF = cash flow in two years = $\$250$

i = annual interest rate = 0.15

n = number of years = 2

Thus,

$$PV = \frac{\$250}{(1 + 0.15)^2} = \frac{\$250}{1.3225} = \$189.04$$



Present Value Concept: Fixed-Payment Loan Terms

- *Simple Loans* require payment of one amount which equals the loan principal plus the interest.

- *Fixed-Payment Loans* are loans where the loan principal and interest are repaid in several payments, often monthly, in equal payment amounts over the loan term.
  - *Installment Loans,* such as car loans and home mortgages are frequently of the fixed-payment type.

Yield to Maturity: Loans

2. Fixed Payment Loan (i = 12%)

$$\$1000 = \frac{\$126}{(1+i)} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \ldots + \frac{\$126}{(1+i)^{25}}$$

$$LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}$$

Yield to Maturity: Bonds

3. Coupon Bond (Coupon rate = 10% = *C*/*F*)

$$P = \frac{\$100}{(1+i)} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \ldots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}$$

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Consol: Fixed coupon payments of $\$C$ forever

$$P = \frac{C}{i} \qquad i = \frac{C}{P}$$

Yield to Maturity: Bonds

4. One-Year Discount Bond (P = \$900, F = \$1000)

$$\$900 = \frac{\$1000}{(1+i)} \Rightarrow$$

$$i = \frac{\$1000 - \$900}{\$900} = .111 = 11.1\%$$

$$i = \frac{F-P}{P}$$

Relationship Between Price and Yield to Maturity

TABLE 3.1 Yields to Maturity on a 10% Coupon Rate Bond Maturing in 10 Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- What’s your findings from above table?

Relationship Between Price and Yield to Maturity

- The value of a bond (price) and YTM are negatively related.

- If *i* increases, the PV of any given cash flow is lower; hence, the price of the bond must be lower.

Current Yield

$$i_c = \frac{C}{P}$$

- Current yield (CY) is just an approximation for YTM – easier to calculate. However, we should be aware of its properties:
  1. If a bond's price is near par and has a long maturity, then CY is a good approximation.
  2. A change in the current yield always signals change in same direction as yield to maturity

Yield on a Discount Basis

$$i_{db} = \frac{(F - P)}{F} \times \frac{360}{(\textit{number of days to maturity})}$$

- One-Year Bill (P = \$900, F = \$1000)

$$i_{db} = \frac{\$1000 - \$900}{\$1000} \times \frac{360}{365} = .099 = 9.9\%$$

https://www.youtube.com/watch?v=kO8x8eoU3L4

- Two Characteristics
  1. Understates yield to maturity; longer the maturity, greater is understatement
  2. Change in discount yield always signals change in same direction as yield to maturity

Break Time - Five Minute University

- https://www.youtube.com/watch?v=kO8x8eoU3L4

Derivation of Demand Curve

- Point A: $P = \$950$ $i = 5.3\%$ $B^d = 100$
- Point B: $P = \$900$ $i = 11.1\%$ $B^d = 200$
- Point C: $P = \$850$ $i = 17.6\%$ $B^d = 300$
- Point D: $P = \$800$ $i = 25.0\%$ $B^d = 400$
- Point E: $P = \$750$ $i = 33.0\%$ $B^d = 500$
- Demand Curve is B^d in Figure 4.1 which connects points A, B, C, D, E.
  - Has usual downward slope

Derivation of Supply Curve

- Point F: $P = \$750$ $i = 33.0\%$ $B^s = 100$
- Point G: $P = \$800$ $i = 25.0\%$ $B^s = 200$
- Point C: $P = \$850$ $i = 17.6\%$ $B^s = 300$
- Point H: $P = \$900$ $i = 11.1\%$ $B^s = 400$
- Point I: $P = \$950$ $i = 5.3\%$ $B^s = 500$
- Supply Curve is B^s that connects points F, G, C, H, I, and has upward slope

Supply and Demand for Bonds

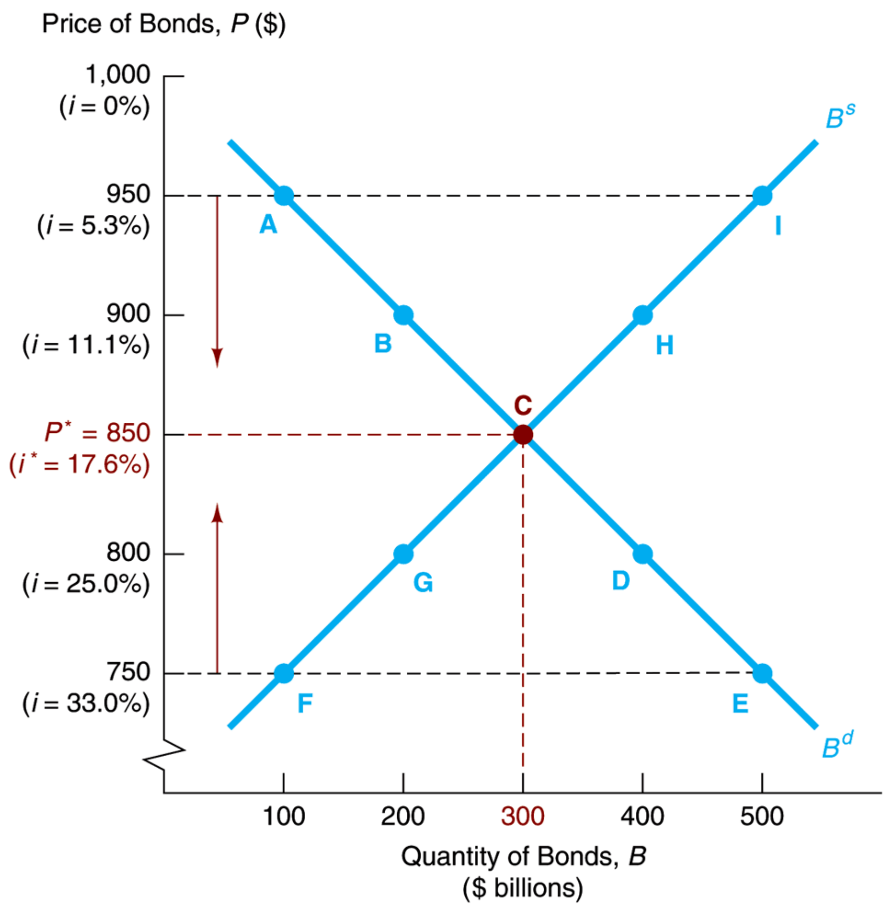


Figure 4.1 Supply and Demand for Bonds

Equilibrium in the bond market occurs at point C, the intersection of the demand curve B^d and the bond supply curve B^s. The equilibrium price is $P^* = \$850$, and the equilibrium interest rate is $i^* = 17.6\%$.

Market Equilibrium

The equilibrium follows what we know from supply-demand analysis:

1. Occurs when $B^d = B^s$, at $P^* = 850$, $i^* = 17.6\%$
2. When $P = \$950$, $i = 5.3\%$, $B^s > B^d$ (excess supply): $P \downarrow$ to P^*, $i \uparrow$ to i^*
3. When $P = \$750$, $i = 33.0$, $B^d > B^s$ (excess demand): $P \uparrow$ to P^*, $i \downarrow$ to i^*

Market Conditions

Market equilibrium occurs when the amount that people are willing to buy (*demand*) equals the amount that people are willing to sell (*supply*) at a given price

Excess supply occurs when the amount that people are willing to sell (*supply*) is greater than the amount people are willing to buy (*demand*) at a given price

Excess demand occurs when the amount that people are willing to buy (*demand*) is greater than the amount that people are willing to sell (*supply*) at a given price

Distinction Between Real and Nominal Interest Rates

- Real Interest Rate (Approximation)

$$i_r = i - \pi^e$$

- If i = 5% and π^e = 0% then

$$i_r = 5\% - 0\% = 5\%$$

- If i = 10% and π^e = 20% then

$$i_r = 10\% - 20\% = -10\%$$

Distinction Between Real and Nominal Interest Rates (cont.)

- Real Interest Rate (accurate way)

(1 + Nominal interest rate) =(1 + Real interest rate)

(1 + Inflation rate)

$$\text{Real interest rate} = \frac{1 + \text{Nominal interest rate}}{1 + \text{Inflation rate}} - 1$$

Difference between the two approaches

Real interest rate = Nominal interest rate - Inflation rate

$$\text{Real interest rate} = \frac{1 + \text{Nominal interest rate}}{1 + \text{Inflation rate}} - 1$$

Nominal Rate = 10%; Inflation Rate = 6%;

Real Rate = ?

r = (1.10/1.06)-1 = 3.8%	r = 10% - 6% = 4%

Real Interest Rates

- Interest rate that is adjusted for expected changes in the price level
- Real interest rate more accurately reflects true cost of borrowing
- When the real rate is low, there are greater incentives to borrow and less to lend
- We usually refer to this rate as the *ex ante* real rate of interest because it is adjusted for the *expected* level of inflation. We can calculate the *ex post* real rate based on the observed level of inflation.

U.S. Real and Nominal Interest Rates

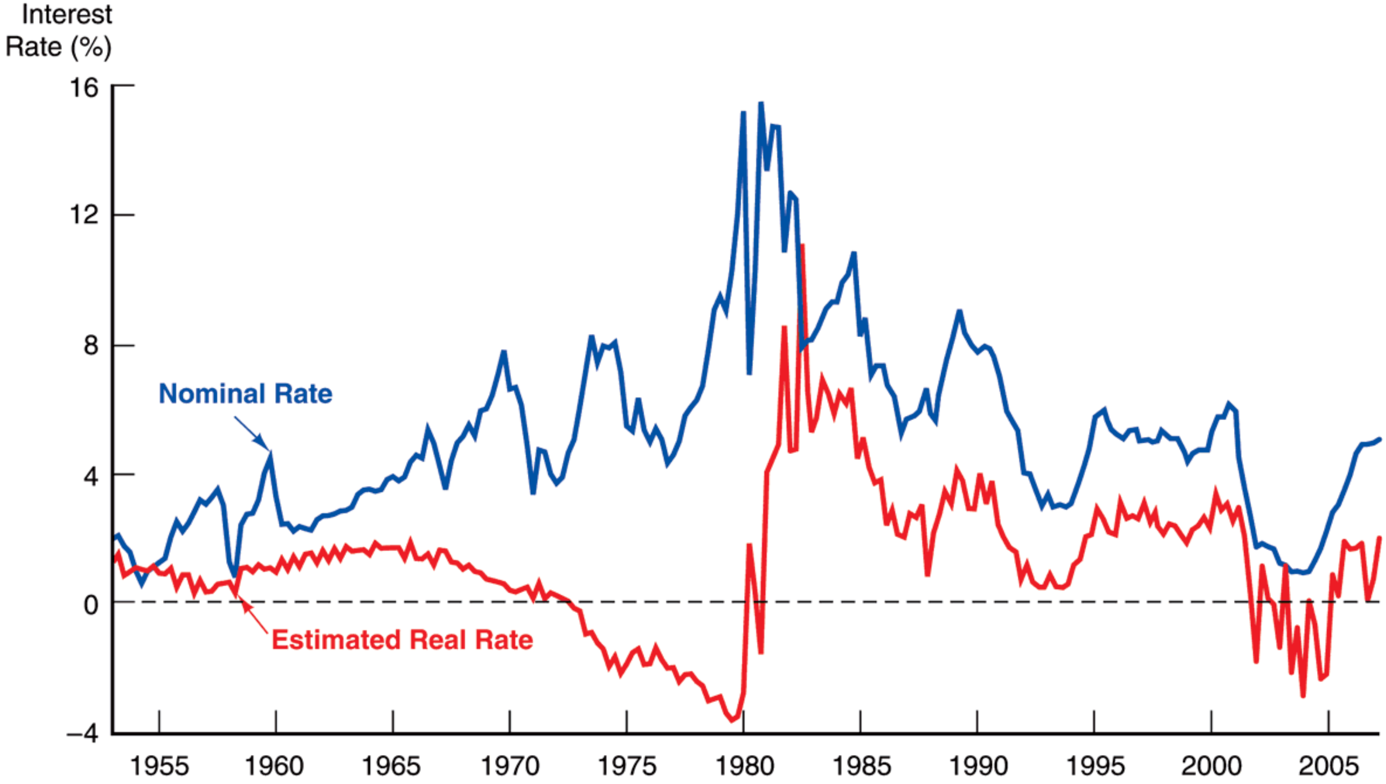


Figure 3.1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2007

Sources: Nominal rates from the Citibase databank. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie–Rochester Conference Series on Public Policy* 15 (1981): 151–200. This involves estimating expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.

Sample of current rates and indexes http://www.martincapital.com/charts.htm

Distinction Between Interest Rates and Returns

- Rate of Return: we can decompose returns into two pieces:

$$\text{Return} = \frac{\text{C} + P_{t+1} - P_t}{P_t} = i_c + g$$

where $i_c = \dfrac{C}{P_t}$ = current yield, and

$$\text{g} = \frac{P_{t+1} - P_t}{P_t}$$ = capital gains.

Key Facts about the Relationship Between Rates and Returns

TABLE 3.2 One-Year Returns on Different-Maturity 10% Coupon Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	− 8.3	+ 1.7
1	10	1,000	1,000	0.0	+10.0

Sample of current coupon rates and yields on government bonds
http://www.bloomberg.com/markets/iyc.html

Maturity and the Volatility of Bond Returns

- Key findings from Table 3.2:

  1. Only bond whose return = yield is one with maturity = holding period
  2. For bonds with maturity > holding period, P ↓ implying capital loss
  3. Longer is maturity, greater is price and return changes
  4. Bond with high initial interest rate can still have negative return

Maturity and the Volatility of Bond Returns (cont.)

- Conclusion from Table 3.2 analysis

    1. Prices and returns more volatile for long-term bonds because have higher interest-rate risk

    2. No interest-rate risk for any bond whose maturity equals holding period

Reinvestment Risk

1. Occurs if hold series of short bonds over long holding period

2. *i* at which reinvest uncertain

3. Gain from *i* ↑, lose when *i* ↓

Summary for today

- Measuring interest rates: we examined several techniques for measuring the interest rate required on debt instruments.

- The distinction between real and nominal interest rates: we examined the meaning of interest in the context of price inflation.

- The distinction between interest rates and returns: we examined what each means and how they should be viewed for asset valuation.

Next Topic

- The Money Market

- Mishkin and Eakins chapter 9

Group Presentation Topics

1. A historic review to the interest rate and/or mortgage rate in your home country from 2000 to present. Explain the causes and consequences of the major fluctuations during the period.
2. An investigation to the performance history of the stock market in your home country. Use some major events to explain whether and how they have affected the stock market index.

Group Presentation Topics (Cont.)

3. A historic review to the exchange rate of your home currency against US dollars from 2006 to 2016. Explain the causes and consequences of the major fluctuations during the period.

4. A case study for a chosen financial institution (for example the largest bank / investment bank / mutual fund / insurance company) in your home country.

Group Presentation Topics (Cont.)

5. A historic review to the bond market in your home country. Please also introduce the types of government bond and corporate bond currently traded.