Modigliani Miller Theorem

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Expected Utility

Assume there are two possible states of nature. Precisely one of them will occur.

x_1 = wealth in state s_1. π_1= probability of state s_1.
x_2 = wealth in state s_2. π_2= probability of state s_2.

Expected utility theory implies that the decision-maker's preferences can be represented by:

$$V\langle x_1, x_2\rangle = \pi_1 u(x_1) + \pi_2 u(x_2).$$

These preferences are additively separable between the states.

Motivation either s_1 or s_2 occurs but not both simultaneously. Hence there is no scope for complementarity between consumption in the two states.

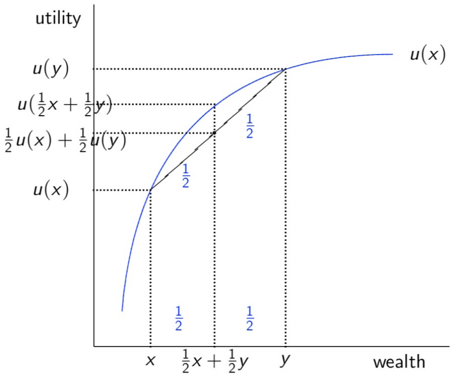
Risk Aversion

Risk attitudes are represented by the curvature of the von-Neumann Morgenstern utility function, u .

- a concave utility function corresponds to risk-aversion
- a convex utility function corresponds to risk-preference (or risk loving behaviour).
- a linear utility function corresponds to risk neutrality.

This is illustrated by the utility of wealth diagram, (on the following slides). An individual has wealth £x with probability $\frac{1}{2}$ or £y also with probability $\frac{1}{2}$.

Expected wealth $= \frac{1}{2}x + \frac{1}{2}y$.


utility
$u(x)$
$u(y)$
$u(\frac{1}{2}x + \frac{1}{2}y)$
$\frac{1}{2}u(x) + \frac{1}{2}u(y)$
$u(x)$
$\frac{1}{2}$
$\frac{1}{2}$
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$\frac{1}{2}$
x
$\frac{1}{2}x + \frac{1}{2}y$
y
wealth

Concave utility

Recall an individual is risk-averse if and only if u is concave.

Definition

u is concave if for all x, y and $\pi : 0 \leq \pi \leq 1$:

$$u(\pi x + (1-\pi)y) \geq \pi u(x) + (1-\pi)u(y).$$

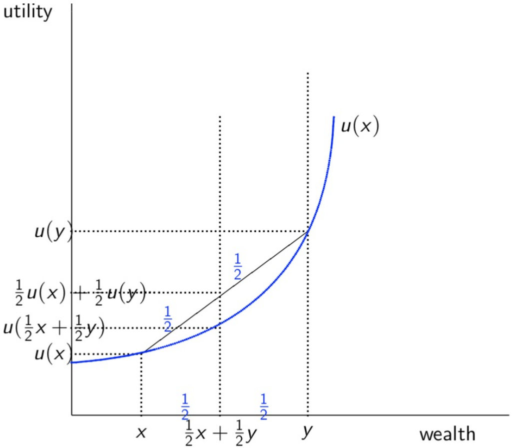
Properties of concave functions

1. u is concave if and only if $u'(x)$ is decreasing in x.
2. u is concave if and only if $u''(x) \leq 0$.

$u'(x)$ decreasing implies that the marginal utility of wealth is decreasing.

A fair gamble is rejected since the possible gains yield less extra utility than the possible losses.

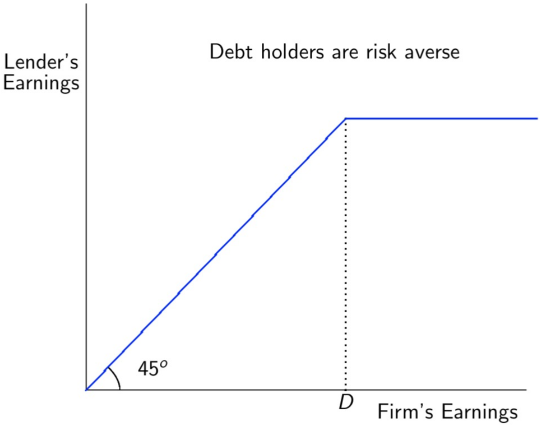
Risk Loving - Convex Utility



Debt and Equity

- Assume that a firm has issued bonds which require it to make (total) payments £D to bondholders. The loan is made under the terms of the standard debt contract.
  - If the firm's returns $R \geqslant D$ then the firm pays D to bondholders.
  - If the firm's returns $R < D$ then the firm pays R to bondholders.
- The only other security issued is equity.
- Equity-holders have limited liability. Hence they cannot be required to contribute additional funds to repay the debt.
- Returns to debt are concave in earnings.
- Returns to equity are a convex function of earnings.

Return to debt



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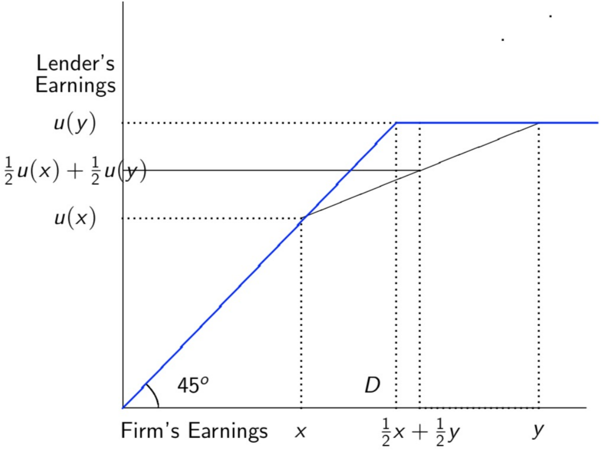
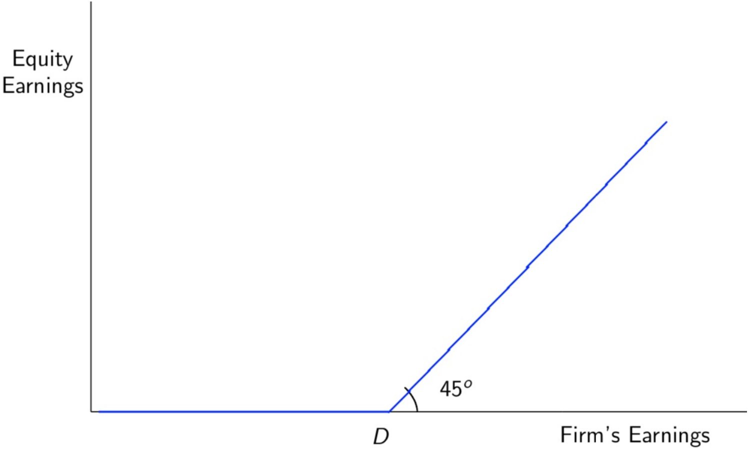


Return to equity



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- Limited liability means that they receive all the rewards of risky investments but their losses are limited.
- However bondholders are risk-averse since their returns are a concave function of the firm's earnings.

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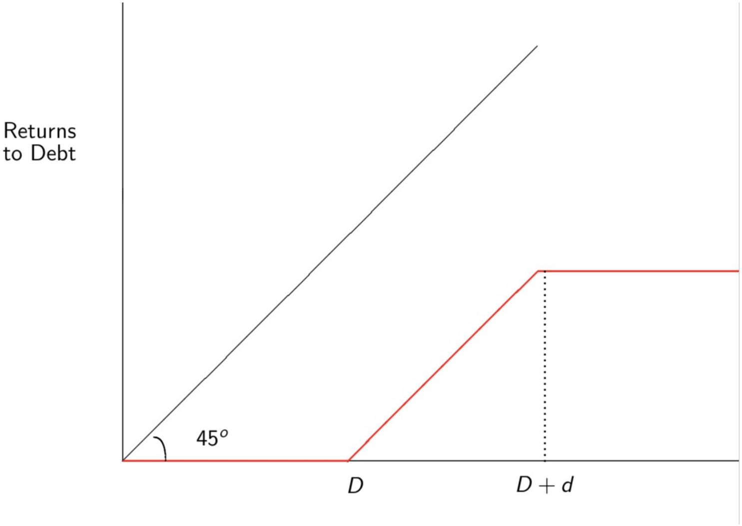
- It subsequently issues junior debt, which has contractual payments of £ d next period.


Returns
to Debt
45°
D
D + d

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- Is debt, which can be exchanged for equity at a predetermined conversion rate. At the discretion of the person holding the security.

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- 2nd Modigliani-Miller Theorem, effects of changes in dividend policy.

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- Returns on debt, 3-5%; Returns on equity, 15-20%.
- The conventional wisdom was that a levered firm was worth more because it had a lower cost of capital.
- In contrast Modigliani and Miller showed that it had the same value as an all equity financed firm.

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- $T_F = S_F + B_F$.

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  - a firm's total returns, X, are unaffected by its financial decisions; investors can borrow and lend on the same terms as firms;
  - then, in equilibrium, the firm's debt-equity ratio cannot affect its value.

proof

Proof Consider two firms, Firm 1 and Firm 2, both of whose earnings may be described by the same random variable X.

Firm 1 is all equity financed, $T_1 = S_1$.

Firm 2 is levered, (i.e. it has issued debt), $T_2 = B_2 + S_2$.

The total payment to shareholders in Firm 2 is, $X - B_2 r$.

$r =$ market interest rate.

Suppose, if possible, the levered firm has higher value, $T_2 > T_1$.

Consider an investor who initially owns fraction α of the equity of Firm 2. This portfolio yields returns $\alpha\left(X - B_2 r\right)$.

- Suppose (s)he sold this portfolio and borrowed £αB_2.
- He/she could buy fraction $\frac{\alpha S_2 + \alpha B_2}{S_1}$ of the equity in Firm 1.
- This gives returns:

$$\frac{\alpha S_2 + \alpha B_2}{S_1} X - \alpha B_2 r$$

$$= \alpha\left(\frac{T_2}{T_1} X - B_2 r\right) > \alpha\left(X - B_2 r\right).$$

- This sequence of trades will give the investor a riskless profit. This cannot happen in equilibrium.
- Thus we may conclude $T_1 \geqslant T_2$.

Now suppose that $T_2 < T_1$.

Consider an investor who initially who owns fraction α of the equity of Firm 1.
Returns $= \alpha X$, Value $= \alpha T_1$.

Suppose instead the individual purchased fraction $\alpha \frac{T_1}{T_2}$ of shares in Firm 2 and $\alpha \frac{T_1}{T_2} B_2$ bonds.

$$\text{Cost} \quad = \alpha \frac{T_1}{T_2} S_2 + \alpha \frac{T_1}{T_2} B_2 = \alpha \frac{T_1}{T_2} (S_2 + B_2) = \alpha \frac{T_1}{T_2} T_2 = \alpha T_1.$$

$$\text{Returns} = \alpha \frac{T_1}{T_2}(X - B_2 r) + \alpha \frac{T_1}{T_2} B_2 r = \alpha \frac{T_1}{T_2} X > \alpha X.$$

Thus the investor has been able to get higher returns at the same cost.

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- Thus all individuals are indi§erent between the situation before and after the firm changes its debt-equity ratio.
- The key point is that portfolios, which yield the same returns, must have the same price in equilibrium.

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- THE OPTIMAL FINANCIAL STRUCTURE OF THE FIRM IS 100% DEBT.