CHAPTER 5

INTRODUCTION TO VALUATION:
TIME VALUE OF MONEY

BASIC DEFINITIONS

- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – "exchange rate" between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

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FUTURE VALUES – EXAMPLE 1

- Suppose you invest $1,000 for one year at 5% per year. What is the future value in one year?
  - Interest = 1,000(.05) = 50
  - Value in one year = principal + interest = 1,000 + 50 = 1,050
  - Future Value (FV) = 1,000(1 + .05) = 1,050
- Suppose you leave the money in for another year. How much will you have two years from now?
  - FV = 1,000(1.05)(1.05) = 1,000$(1.05)^2$ = 1,102.50

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FUTURE VALUES: GENERAL FORMULA

- $FV = PV(1 + r)^t$
  - FV = future value
  - PV = present value
  - r = period interest rate, expressed as a decimal
  - t = number of periods
- Future value interest factor = $(1 + r)^t$

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EFFECTS OF COMPOUNDING

- Simple interest vs. Compound interest
- Consider the previous example
  - FV with simple interest = 1,000 + 50 + 50 = 1,100
  - FV with compound interest = 1,102.50
  - The extra 2.50 comes from the interest of .05(50) = 2.50 earned on the first interest payment

FUTURE VALUES – EXAMPLE 2

- Suppose you invest the $1,000 from the previous example for 5 years. How much would you have?
  - 5 N; 5 I/Y; 1,000 PV
  - CPT FV = -1,276.28
- The effect of compounding is small for a small number of periods, but increases as the number of periods increases. (Simple interest would have a future value of $1,250, for a difference of $26.28.)

FUTURE VALUES – EXAMPLE 3

- Suppose you had a relative deposit $10 at 5.5% interest 200 years ago. How much would the investment be worth today?
  - 200 N; 5.5 I/Y; 10 PV
  - CPT FV = -447,189.84
- What is the effect of compounding?
  - Simple interest = 10 + 200(10)(.055) = 120.00
  - Compounding added $447,069.84 to the value of the investment

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FUTURE VALUE AS A GENERAL GROWTH FORMULA

- Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you sell 3 million widgets in the current year, how many widgets do you expect to sell in the fifth year?
  - 5 N;15 I/Y; 3,000,000 PV
  - CPT FV = -6,034,072 units
    (remember the sign convention)

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PRESENT VALUES

- How much do I have to invest today to have some amount in the future?
  - $FV = PV(1 + r)^t$
  - Rearrange to solve for $PV = FV / (1 + r)^t$
- When we talk about discounting, we mean finding the present value of some future amount.
- When we talk about the “value” of something, we are talking about the present value unless we specifically indicate that we want the future value.

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PRESENT VALUE – ONE PERIOD EXAMPLE

- Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?
- $PV = 10,000 / (1.07)^1 = 9,345.79$
- Calculator
  - 1 N
  - 7 I/Y
  - 10,000 FV
  - CPT PV = -9,345.79

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PRESENT VALUES – EXAMPLE 2

- You want to begin saving for your daughter's college education and you estimate that she will need $150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?
  - N = 17; I/Y = 8; FV = 150,000
  - CPT PV = -40,540.34 (remember the sign convention)

PRESENT VALUES – EXAMPLE 3

- Your parents set up a trust fund for you 10 years ago that is now worth $19,671.51. If the fund earned 7% per year, how much did your parents invest?
  - N = 10; I/Y = 7; FV = 19,671.51
  - CPT PV = -10,000

PRESENT VALUE – IMPORTANT RELATIONSHIP I

- For a given interest rate – the longer the time period, the lower the present value
  - What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%
  - 5 years: N = 5; I/Y = 10; FV = 500
    CPT PV = -310.46
  - 10 years: N = 10; I/Y = 10; FV = 500
    CPT PV = -192.77

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PRESENT VALUE – IMPORTANT RELATIONSHIP II

- For a given time period – the higher the interest rate, the smaller the present value
  - What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?
    - Rate = 10%: N = 5; I/Y = 10; FV = 500
      CPT PV = -310.46
    - Rate = 15%; N = 5; I/Y = 15; FV = 500
      CPT PV = -248.59

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THE BASIC PV EQUATION - REFRESHER

- $PV = FV / (1 + r)^t$
- There are four parts to this equation
  - PV, FV, r and t
  - If we know any three, we can solve for the fourth
- If you are using a financial calculator, be sure to remember the sign convention or you will receive an error (or a nonsense answer) when solving for r or t

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DISCOUNT RATE

- Often we will want to know what the implied interest rate is on an investment
- Rearrange the basic PV equation and solve for r
  - $FV = PV(1 + r)^t$
  - $r = (FV / PV)^{1/t} – 1$
- If you are using formulas, you will want to make use of both the y^x and the 1/x keys

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DISCOUNT RATE – EXAMPLE 1

- You are looking at an investment that will pay $1,200 in 5 years if you invest $1,000 today. What is the implied rate of interest?
  - r = $(1,200 / 1,000)^{1/5} - 1 = .03714 = 3.714\%$
  - Calculator – the sign convention matters!!!
    - N = 5
    - PV = -1,000 (you pay 1,000 today)
    - FV = 1,200 (you receive 1,200 in 5 years)
    - CPT I/Y = 3.714%

DISCOUNT RATE – EXAMPLE 2

- Suppose you are offered an investment that will allow you to double your money in 6 years. You have 10,000 to invest. What is the implied rate of interest?
  - N = 6
  - PV = -10,000
  - FV = 20,000
  - CPT I/Y = 12.25%

DISCOUNT RATE – EXAMPLE 3

- Suppose you have a 1-year old son and you want to provide $75,000 in 17 years towards his college education.
  - You currently have $5,000 to invest.
  - What interest rate must you earn to have the $75,000 when you need it?

    - N = 17; PV = -5,000; FV = 75,000

    - CPT I/Y = 17.27%

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FINDING THE NUMBER OF PERIODS

- Start with the basic equation and solve for t (remember your logs)
  - $FV = PV(1 + r)^t$
  - $t = \ln(FV / PV) / \ln(1 + r)$

- You can use the financial keys on the calculator as well; just remember the sign convention.

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NUMBER OF PERIODS – EXAMPLE 1

- You want to purchase a new car, and you are willing to pay $20,000.
  - If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?
    - I/Y = 10; PV = -15,000; FV = 20,000
    - CPT N = 3.02 years

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NUMBER OF PERIODS – EXAMPLE 2

- Suppose you want to buy a new house.
  - You currently have $15,000, and you figure you need to have a 10% down payment plus an additional 5% of the loan amount for closing costs.
  - Assume the type of house you want will cost about $150,000 and you can earn 7.5% per year.
  - How long will it be before you have enough money for the down payment and closing costs?

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NUMBER OF PERIODS – EXAMPLE 2 CONTINUED

- How much do you need to have in the future?
  - Down payment = .1(150,000) = 15,000
  - Closing costs = .05(150,000 – 15,000) = 6,750
  - Total needed = 15,000 + 6,750 = 21,750
- Compute the number of periods
- Using a financial calculator:
  - PV = -15,000; FV = 21,750; I/Y = 7.5
  - CPT N = 5.14 years
- Using the formula:
  - t = ln(21,750 / 15,000) / ln(1.075) = 5.14 years

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