Corporate Finance

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Session 2 - Time Value of Money

UNIVERSITY OF CAMBRIDGE
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Roadmap

- Calculating Present Values
- Annuities & Perpetuities
- Applying these formulas in real life.

Present and Future Values

- Present and Future Values:
  - Compounding a single lump sum.
  - Discounting a single lump sum.
- Compounding more than once a year.
- Compounding and discounting many cash flows.

Present and Future Values: Part I

- Discounting a single lump sum:

$$PV_t = \frac{C}{(1+r)^t}$$

- Compounding a single lump sum:

$$FV_t = C * (1+r)^t$$

- Annuity: PV of fixed amount C for t periods:

$$PV_t = \frac{C}{r} \times \left(1 - \frac{1}{(1+r)^t}\right)$$

- Future value of a fixed amount C at the end of t periods:

$$FV_t = \frac{C}{r} \times \left((1+r)^t - 1\right)$$

Present and Future Values: Part II

- Perpetuity: PV of fixed amount C forever:

$$PV_t = \frac{C}{r}$$

- Annuity: PV of fixed amount C for t periods with growth:

$$PV_t = \frac{C}{r-g} \times \left(1 - \left(\frac{1+g}{1+r}\right)^t\right)$$

Example I

- Suppose you owe $2000 on a VISA card, and the interest rate is 2% per month. If you make the minimum monthly payments of $50, how long will it take you to pay it off?

Example II

- You have just won the lottery. The payoff is a million dollars. Except the lottery promises to pay you $100,000 every year for the next 10 years. If the discount rate is 8%, what is the real value of your prize?

Example III: Cheap Financing and Rebate

- TF Banks are making 10% car loans. Should you choose the 7.59% financing from the car company or should you borrow the money from the bank and go for the $500 rebate?

Example III: Cheap Financing and Rebate

- Assuming no down payment, 36 month loan and monthly compounding:
- **Bank**: PV = \$61,000, $r = \frac{.10}{12} = 0.833\%$ p.m., t = 36.

$$61,000 = \frac{C}{(0.833\%)} \times \left(1 - \frac{1}{(1+0.833\%)^{36}}\right) \Rightarrow C = 1968.29$$

- **Car company**: PV = **\$61,500**, $r = \frac{.0759}{12} = 0.6325\%$ p.m., t = 36.

$$61,000 = \frac{C}{(0.6325\%)} \times \left(1 - \frac{1}{(1+0.6325\%)^{36}}\right) \Rightarrow C = 1915.58$$

- Cheaper to go with the financing option!

Example IV: Kangaroo vs Turtle

- Kangaroo Autos is offering free credit on a new $10,000 car. You pay $1,000 down and then $300 a month for the next 30 months.
- Turtle Motors next door does not offer free credit but gives you $1,000 off the list price. If the rate of interest is 10%, APR

  which company is offering the better deal?
- Kangaroo Motors costs:

$$PV = 1,000 + \frac{300}{\left(\frac{0.1}{12}\right)} \times \left(1 - \frac{1}{(1+\frac{0.1}{12})^{30}}\right) \Rightarrow PV = 8,934.11$$

- Kangaroo is cheaper than the $9,000. Here we compare PVs directly.

Example V: Annuities and Perpetuities with Growth

- You own an oil pipeline which will generate a $2 million cash return over the coming year. Operating costs are negligible and it is expected to last for a very long time.
- Unfortunately, the volume of oil shipped is declining and cash flows are expected to decline by four percent per year. The discount rate is 10%.
- Question A: What is the present value of the pipeline's cash flows if its cash flows are expected to last forever?
- Question B: What is the present value of the cash flows if the pipeline is scrapped after 20 years?

Example V: Annuities and Perpetuities with Growth

- Part A requires a perpetuity with growth.
- We have CF=\$2,000,000; r=10%; g=**–4%.**

$$PV = \frac{2,000,000}{0.1 - 0.04} = \$14,285,714.$$

- Part B requires an annuity with growth.
- We have CF=\$2,000,000; r=10%; g=–4%; t=20 years.

$$PV = \frac{2,000,000}{0.1 - 0.04} \times \left(1 - \left(\frac{1 - 0.04}{1 + 0.10}\right)^{20}\right) \Rightarrow PV = 13,347,131.$$

Example VI: Annuities and Perpetuities with Growth

- You have just read an advertisement reading: “Pay us \$100 a year for 10 years and we will pay you \$100 a year thereafter in perpetuity”.
- If this is a fair deal, what is the rate of interest?
- We have an annuity right now for 10 years and a perpetuity after that.

$$\frac{100}{r} \times \frac{1}{(1+r)^{10}} = \left[\frac{100}{r} \times \left(1 - \left(\frac{1}{1+r}\right)^{10}\right)\right] \Rightarrow$$

$$1 = \frac{2}{(1+r)^{10}} \Rightarrow r = 2^{\frac{1}{10}} - 1 \Rightarrow r = 7.17\%$$